Geometry

Pacing Guide and Unpacked Standards

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Groveport Madison Math Pacing Guide- Geometry

➢ **Indicates Blueprint Focus Standards**
* **Indicates Modeling Standard**

Geometry	**Congruence and Proof**	**Similarity, Right Triangles, & Trigonometry**	**Circles**	**Probability**	**Standards for Mathematical Practice**
1st 9 Wks	➢**G.CO.1** Know definitions of ray,angle,line,arc length ➢**G.CO.9** Prove and apply theorems about lines and angles ➢**G.CO.10** Prove and apply triangle theorems ➢**G.CO.12** Make formal geometric constructions ➢**G.GPE.6** Find a point on a line partitioning a segment given a ratio ➢**G.GPE.7** Compute perimeters on coordinate plane	➢**G.MG.1*** Use properties of shapes to describe objects ➢**G.MG.3*** Apply geometric methods to solve problems	➢**G.GMD.1** Give informal argument for formulas for circles cylinder, cones, ➢**G.GMD.3*** Use volume formulas for cylinders, pyramids, cones, spheres ➢**G.GMD.4** Identify cross sections of 3-D shapes ➢**G.GMD.5** Changing measures affect similarity ➢**G.GMD.6** Scaling by a factor of k, ➢**G.CO.9** Prove and apply theorems lines and angles (radius, diameter,circumference)		**MP.1** Make sense of problems and persevere in solving them **MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the reasoning of others **MP.4** Model with mathematics **MP.5** Use appropriate tools strategically **MP.6** Attend to precision **MP.7** Look for and make use of structure **MP.8** Look for and express regularity in repeated reasoning
2nd 9 Wks	➢**G.CO.7** Show triangles are are congruent iff corresponding parts are congruent ➢**G.CO.8** Explain triangle congruence through transformations ➢**G.CO.10** Prove and apply theorems about triangles ➢**G.GPE.4** Prove geometric theorems on coordinate plane ➢**G.GPE.5** Justify slope criteria for parallel/perpendicular lines given a point	➢**G.MG.1*** Use properties of shapes to describe objects ➢**G.MG.3*** Apply geometric methods to solve problems ➢**G.SRT.4** Prove and apply theorems about triangles ➢**G.SRT.5** Justify relationships in figures using triangles (interior angle sum)			

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Geometry	Congruence and Proof	Similarity, Right Triangles, & Trigonometry	Circles	Probability	Standards for Mathematical Practice
3rd 9 Wks	➢G.CO.11 Prove and apply theorems about parallelograms	➢G.SRT.1 Identify properties of dilation ➢G.SRT.2 Decide if shapes are similar by corresponding parts ➢G.SRT.3 Establish AA similarity ➢G.SRT.6 Prove and apply theorems about triangles ➢G.SRT.7 Explain and use properties of sine and cosine ➢G.SRT.8* Solve problems using trigonometry and Pythagorean theorem ➢G.MG.1* Use properties of shapes to describe objects ➢G.MG.3* Apply geometric methods to solve problems ➢G.GMD.6 Understand the scale factor affects area/volume			MP.1 Make sense of problems and persevere in solving them MP.2 Reason abstractly and quantitatively MP.3 Construct viable arguments and critique the reasoning of others MP.4 Model with mathematics MP.5 Use appropriate tools strategically MP.6 Attend to precision MP.7 Look for and make use of structure MP.8 Look for and express regularity in repeated reasoning
4th 9 Wks	➢G.CO.1 Know definition of circle ➢G.CO.2 Represent transformations ➢G.CO.3 Identify line/rotational symmetry ➢G.CO.4 Develop definitions of rotations, reflections, translations ➢G.CO.5 Draw a transformed figure ➢G.CO.6 Predict the effect of a transformation ➢G.CO.13 Construct a triangle, square, hexagon inscribed in a circle G.CO.14 Classify 2 dimensional shapes G.GPE.2 Derive the equation of a parabola G.GPE.3 Derive the equations of an ellipse and hyperbola	➢G.MG.1* Use properties of shapes to describe objects ➢G.MG.2* Apply concepts of density ➢G.MG.3* Apply geometric methods to solve problems G.SRT.9 Derive A=1/2ab sin C G.SRT.10 Explain the proof of the Law of Sines and Cosines G.SRT.11 Understand and apply the Laws of Sines and Cosines G.GMD.2 Give an argument using Cavalieri's principle G.GMD.5 Understand how and why changes of measure result in similar and non-similar shapes	➢G.C.1 Prove all circles are similar ➢G.C.2 Identify and describe angles, radii, chords, tangents,arcs ➢G.C.3 Construct inscribed and circumscribed circles of a triangle ➢G.C.5 Find arc length and sector area G.C.4 Construct a tangent line ➢G.GPE.1 Derive the equation of a circle	➢S.CP.1* Describe events as subsets of sample space ➢S.CP.2* Understand when 2 events are independent ➢S.CP.3* Understand conditional probability ➢S.CP.4* Construct/interpret frequency tables ➢S.CP.5* Recognize/explain conditional probability in everyday language ➢S.CP.6* Find the conditional probability ➢S.CP.7* Apply/interpret the Addition Rule	

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.CO.1

Know the precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of a point, line, distance along a line, and arc length.

Essential Understanding

Studentswill be expected to be able to describe undefined terms: point line, distance along a line in a plane, circle and distance around a circular arc.

Students will be expected to be able to define perpendicular lines, parallel lines, line segments, and angles.

Students define angles, circles, perpendicular lines, parallel lines, and line segments precisely using the undefined terms.

Extended Understanding

Students can be asked to draw/construct figure(s) and then asked to justify how it (they) meet(s) the definition of the term(s).

Academic Vocabulary/Language

angle
ray
coordinate distance
perpendicular lines
arc/arc length
draw
plane
circle
line segment
point
construct
parallel lines
undefined terms
accurately
definition
defined terms
describe

Essential Skills

- I can define the following terms: angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc
- I understand and can use definitions of content vocabulary to accurately describe figures and relationships among figures

Instructional Strategies

Review vocabulary associated with transformations (e.g. point, line, segment, angle, circle, polygon, parallelogram, perpendicular, rotation reflection, translation).

*Students can be asked to d*raw/construct figure(s) and then asked to justify how it (they) meet(s) the definition of the term(s). For example: *Construct the following: an angle, circle, perpendicular lines, parallel lines, line segment; secondly, explain/justify how the construction(s) meet the definition of the term.*

Common Misconceptions and Challenges

Some students are uncomfortable talking and thinking about "undefined terms." The view is that undefined objects do not so much exist in themselves as they are determined by the properties hypothesized for them; we, thereby, build a foundation for common language in geometry.

Performance Level Descriptors

Limited Recognize definitions of angle, circle, perpendicular lines, parallel lines, and line segment

Basic Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment

Proficient Recognize precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment based on the undefined notions of point, line, distance along a line and distance around a circular arc

Advanced N/A

Accelerated N/A

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.CO.2, G.CO.3

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angel to those that do not (e.g., translations versus horizontal stretch).

3. Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.
   a. Identify figures that have line of symmetry; draw and use lines of symmetry to analyze properties of shapes.
   b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes

Essential Understanding

Students are expected to learn the academic language of the parameters of transformations (rotations, reflections, and translations).

Students will need to be able to articulate what differentiates rigid motions from non-rigid motions.

Extended Understanding

Students can explore/ create: a square from two points on a line; students can determine a way to construct an octagon inscribed in a circle.

Academic Vocabulary/Language

coordinate distance
dilation
reflection
rigid motion
rotation
stretch
transformation
translation
compare
articulate
create
describe
represent

Essential Skills

- I can use various technologies such as transparencies, geometry software, interactive whiteboards, and digital visual presenters to represent and compare rigid and size transformations of figures in a coordinate plane---comparing transformations that preserve distance and angle to those that do not.
- Describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, to include translations and horizontal and vertical stretching.
- Describe the rotations and reflections of a rectangle, parallelogram, trapezoid, or regular polygon that maps each figure onto itself.

Instructional Strategies

Provide students with a pre-image and a final, transformed image, and ask them to describe the steps generate the final image. Show examples with more than one answer(e.g., a reflection might result in the same image as a translation).

Provide both individual and small-group activities, allowing adequate time for students to explore and verify conjectures about transformations and develop precise definitions ofrotations, reflections and translations.

Provide real-world examples of rigid motions (e.g. Ferris wheels for rotation; mirrors for reflection; moving vehicles for translation). Use graph paper, transparencies, tracing paper or dynamic geometry software to obtain images of a given figure under specified transformations.

Emphasize the importance of understanding a transformation as the correspondence between initial and final points, rather than the physical motion.

Use a variety of means to represent rigid motions, including physical manipulatives, coordinate methods, and dynamic geometry software.

Make the transition from transformations as physical motions to functions that take points in the plane as inputs and give other points as outputs. The correspondence between the initial and final points determines the transformation.

Analyze various figures (e.g. regular polygons, folk art designs or product logos) to determine which rotations and reflections carry (map) the figure onto itself. These transformations are the “symmetries” of the figure.

Common Misconceptions and Challenges

The terms “mapping” and “under” are used in special ways when studying transformations. A translation is a type of transformation that moves all the points in the object in a straight line in the same direction.

Students should know that not every transformation is a translation. Students sometimes confuse the terms “transformation” and “translation.”

Performance Level Descriptors

Basic N/A

Limited N/A

Proficient Describe a sequence of transformations that will carry a given figure onto itself or another polygon (G.CO.3)

Accelerated N/A

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.CO.4, G.CO.5

Experiment with Transformations in the Plane

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflections, or translations, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Essential Understanding

Students are expected to know the definitions of translation, reflection, rotation and dilation.

Students are expected to use symbolism to perform and describe transformations and dilations.

Students are expected to graph pre-image and image given a symbolic description.

Extended Understanding

Students can use their knowledge of geometric properties to solve problem; for example, students can explore the relationship between radii of inscribed circles and circumscribed circles of right triangles

Academic Vocabulary/Language

-image
-pre-image
-reflection
-rotation
-transformation
-translation

Essential Skills

- I can develop the definitions of each transformation (rotations, reflections, translations) in regards to the characteristics between pre-image and image points.
- I can transform a geometric figure given a rotation, reflection, or translation.
- I can create sequences of transformations that map a geometric figure onto itself and another geometric figure.
- I can predict and verify the sequence of transformations (a composition) that will map a figure onto another.

Instructional Strategies

Provide both individual and small-group activities, allowing adequate time for students to explore and verify conjectures about transformations and develop precise definitions ofrotations, reflections and translations.

Provide real-world examples of rigid motions (e.g. Ferris wheels for rotation; mirrors for reflection; moving vehicles for translation). Use graph paper, transparencies, tracing paper or dynamic geometry software to obtain images of a given figure under specified transformations.

Provide students with a pre-image and a final, transformed image, and ask them to describe the steps required to generate the final image. Show examples with more than oneanswer (e.g., a reflection might result in the same image as a translation). Work backwards to determine a sequence of transformations that will carry (map) one figure onto another of the same size and shape.

Focus attention on the attributes (e.g. distances or angle measures) of a geometric figure that remain constant under various transformations. Make the transition from transformations as physical motions to functions that take points in the plane as inputs and give other points as outputs. The correspondence between the initial and final points determines the transformation.

Analyze various figures (e.g. regular polygons, folk art designs or product logos) to determine which rotations and reflections carry (map) the figure onto itself. These transformations are the "symmetries" of the figure. Emphasize the importance of understanding a transformation as the correspondence between initial and final points, rather than the physical motion.

Common Misconceptions and Challenges

Students need to know how to graph/recognize lines and equations of lines to perform reflections. Students often assume rotations are about the origin, when they can be about any specified point.

Performance Level Descriptors

Limited Given a geometric figure and a rotation, reflection, or translation, identify the transformed figure (G.CO.5)

Basic N/A

Proficient Describe a sequence of transformations that will carry a given figure onto itself or another polygon (G.CO.5)

Accelerated N/A

Advanced Specify, using abstract values, a sequence of transformations that will carry a given figure onto another (G.CO.4)

Ohio's Learning Standards – Clear Learning Targets Geometry

G.CO.6, G.CO.7, G.CO.8

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of given rigid motions on a given figure; given two figures, use the definitions of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding parts of sides and corresponding pairs of angles are congruent.

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Essential Understanding

Students should know that two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition.

Students should know that for triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles.

Extended Understanding

Students will learn that once triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, this can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Academic Vocabulary/Language

-carries onto
-congruence
-coordinate plane
-corresponding
-equality
-maps onto
-rigid motion
-transformation
-criteria
-develop
-predict
-definition
-explain
-persevere

Essential Skills

- I can use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.
- I can, knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent.
- I can use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- I can use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent; I can explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions; I can use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS.

Instructional Strategies

Make the transition from transformations as physical motions to functions that take points in the plane as inputs and give other points as outputs. The correspondence between the initial and final points determines the transformation.

Analyze various figures (e.g. regular polygons, folk art designs or product logos) to determine which rotations and reflections carry (map) the figure onto itself. These transformations are the "symmetries" of the figure.

Emphasize the importance of understanding a transformation as the correspondence between initial and final points, rather than the physical motion. Use a variety of means to represent rigid motions, including physical manipulatives, coordinate methods, and dynamic geometry software.

Common Misconceptions and Challenges

Students need to know how to graph/recognize lines and equations of lines to perform reflections.

Students often assume rotations are about the origin, when they can be about any specified point.

Students think SSA is a congruence criterion for triangles.

Performance Level Descriptors

Limited N/A

Basic Use geometric descriptions of rigid motions to transform figures (G.CO.6)

Given two triangles, determine if the two triangles are congruent based upon sides and angles (G.CO.7)

Proficient Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent (G.CO.6)

Describe a sequence of transformations that will carry a given figure onto itself or another polygon (G.CO.7)

Accelerated Specify, using numeric values, a sequence of transformations that will carry a given figure onto another (G.CO.6)

Advanced Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent (G.CO.7)

Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions (G.CO.8)

Ohio's Learning Standards – Clear Learning Targets Geometry

G.CO.9, G.CO.10, G.CO.11

9. Prove and appply theorems about lines and angles. Theorems include but are not restricted to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

11. Prove theorems about parallelograms. Theorems include but are not restricted to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Essential Understanding

Students will be able to prove theorems involving lines and angles.
Students will be able to prove that the sum of the angles in a triangle is always 180 degrees

Extended Understanding

Students will understand the basics of parallelograms and be able to prove various theorems related to them.

Academic Vocabulary/Language

- vertical angles
- transversal
- alternate interior angles
- perpendicular bisector
- parallelograms
- prove

Essential Skills

- I can identify and use properties of perpendicular bisector.
- I can identify and use properties of equidistant from endpoint.
- I can identify and use properties of all angle relationships.
- I can prove vertical angles are congruent.
- I can identify the hypothesis and conclusion of a triangle sum theorem.
- I can identify the hypothesis and conclusion of a base angle of isosceles triangles.
- I can identify the hypothesis and conclusion of mid-segment theorem.
- I can classify types of quadrilaterals.
- I can explain theorems for various parallelograms involving opposite sides and angles and relate to figure.
- I can use properties of special quadrilaterals in a proof.

Instructional Strategies

Classroom teachers and mathematics education researchers agree that students have a hard time learning how to do geometric proofs. "Geometry and Proof" is an article by Battista and Clements (1995) that provides information for teachers to help students who struggle learn to do proof. The most significant implication for instructional strategies for proof is stated in their conclusion.

> "Ironically, the most effective path to engendering meaningful use of proof in secondary school geometry is to avoid formal proof for much of students' work. By focusing instead on justifying ideas while helping students build the visual and empirical foundation for higher levels of geometric thought, we can lead students to appreciate the need for formal proof. Only then will we be able to use it meaningfully as a mechanism for justifying ideas."

The article and ideas from Niven (1987) offers a few suggestions about teaching proof in geometry:

- Initial geometric understandings and ideas should be taught "without excessive emphasis on rigor." Develop basic geometric ideas outside an axiomatic framework, and then let the importance of the framework (and the framework itself) emerges from the geometry.
- Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between geometric objects should be central to any geometric study and certainly to proof. Battista and Clement make a powerful argument that the use of dynamic geometry software can be an important tool for helping students understand proof.
- "[A]void the deadly elaboration of the obvious" (Niven, p. 43). Often textbooks begin the treatment of formal proof with "easy" proofs, which appear to students to need no proof at all. After presenting many opportunities for students to "justify" properties of geometric figures, formal proof activities should begin with non-obvious conjectures.
- Use the history of geometry and real-world applications to help students develop conceptual understandings before they begin to use formal proof.

Proofs in high school geometry should not be restricted to the two-column format. Most proofs at the college level are done in paragraph form, with the writer explaining and defending a conjecture. In many cases, the two-column format can hinder the student from making sense of the geometry by paying too much attention to format rather than mathematical reasoning.
Some of the theorems listed in this cluster (e.g. the ones about alternate interior angles and the angle sum of a triangle) are logically equivalent to the Euclidean parallel postulate, and this should be acknowledged.

Use dynamic geometry software to allow students to make conjectures that can, in turn, be formally proven. For example, students might notice that the base angles of an isosceles triangle always appear to be congruent when manipulating triangles on the computer screen and could then engage in a more formal discussion of why this occurs.

Common Core Standards Appendix A states, "Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words.

Common Misconceptions and Challenges

Research over the last four decades suggests that student misconceptions about proof abound:

- even after proving a generalization, students believe that exceptions to the generalization might exist;
- one counterexample is not sufficient;
- the converse of a statement is true (parallel lines do not intersect, lines that do not intersect are parallel); and
- a conjecture is true because it worked in all examples that were explored.

Each of these misconceptions needs to be addressed, both by the ways in which formal proof is taught in geometry and how ideas about “justification” are developed throughout a student’s mathematical education.

Performance Level Descriptors

Limited N/A

Basic Complete a proof of a theorem about lines and angles, triangles, or parallelograms by identifying one or two statements or reasons missing from the proof (G.CO.9-11)

Proficient Complete a proof of a theorem about lines and angles, triangles, or parallelograms requiring a routine proof (G.CO.9-11)

Accelerated Complete a proof of a theorem about lines and angles, triangles, or parallelograms requiring a routine proof (G.CO.9-11)

Advanced Construct a logical formal proof of a theorem about lines and angles, triangles, or parallelograms (G.CO.9-11)

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.CO.12, G.CO.13, G.CO.14

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflectivedevices, paper folding, dynamic geometric software, etc.). Copying a segment: copying an angle; bisecting a segment; constructingperpendicular lines, including the perpendicular bisector of a line segment and constructing a line parallel to a given line through a point not on the line.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribe in a circle.

14. Classify two dimensional figures in a hierarchy based on properties.

Essential Understanding

Students should be able to apply definitions, properties, theorems about line segments, rays, and angles to support geometric constructions.

Student should be able to apply properties, theorems about parallel and perpendicular lines to support geometric constructions.

Students should be able to construct a square, equilateral triangle, regular hexagon inscribed in a circle.

Extended Understanding

Students can create drawings using nothing more than a compass and straightedge: e.g., stars inside of a circle, dodecagons; students can then calculate each inscribed image.

Academic Vocabulary/Language

arc
bisector
circle
circumference
congruent diameter
equilateral
triangle
inscribe
parallel
perpendicular radius
regular hexagon
regular polygon
square
straightedge
triangle
compass

Essential Skills

- I can copy: a segment, an angle.
- I can bisect: a segment, an angle.
- I can construct perpendicular lines, including the perpendicular bisector of a line segment.
- I can construct a line parallel to a given line through a point not on the line.
- I can construct an equilateral triangle so that each vertex of the triangle is on the circle.
- I can construct a square so that each vertex of the square is on the circle.
- I can construct a regular hexagon so that each vertex of the regular hexagon is on the circle.

Instructional Strategies

Students should analyze each listed construction in terms of what simpler constructions are involved (e.g., constructing parallel lines can be done with two different construction of perpendicular lines).

Challenge students to perform the same construction using a compass and a string. Use paper folding to produce a reflection; use bisections to produce reflections. Ask students to produce "how to" manuals, giving verbal instructions for particular constructions.
Provide meaningful opportunities (constructing the centroid or the incenter of a triangle) to offer students practice in executing basic constructions.

Compare dynamic geometry commands to sequences of compass-and-straightedge steps. Utilize technology in construction activities.

To ensure that students are correctly making constructions and not just estimating a parallel line or the bisector of an angle, remind students that you will be looking for the marks made by the sharp points of the compass and that there should be arcs made of the drawing; it should be clear where the arcs cross each other.

Common Misconceptions and Challenges

Some students believe that construction is the same as sketching or drawing.

Teachers should emphasize the need for precision and accuracy when doing constructions. Stress the ideas that a compass and straightedge are identical to a protractor and ruler. Explain the definition of measurement and construction.

If not using safety compasses, make certain that students know to use tool in a cautious, safe manner.

Remind students to keep compass opened at the same setting throughout the entire construction unless they are told to readjust the tool.

Performance Level Descriptors

Limited N/A

Basic Identify the step or steps needed to complete a given construction using a variety of tools and methods (G.CO.12)

Proficient Make formal geometric constructions with a variety of tools and methods. (G.CO.12)

Advanced N/A

Accelerated Construct an equilateral triangle, a square, and a regular hexagon inscribe in a circle (G.CO.13)

Ohio's Learning Standards – Clear Learning Targets Geometry

G.GPE.1-2	Essential Understanding	Academic Vocabulary/Language
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 2. Derive the equation of a parabola given a focus and directrix.	Students will derive the equation of a circle and understand the horizontal and vertical shifts (h, k). Students will understand how to determine the radius of a circle when given its equation. **Extended Understanding** Students will be able to derive the equation of a parabola when given its focus and directrix.	- focus - directrix - parabola - complete the square - circle - derive

Essential Skills

- I can define a circle.
- I can use Pythagorean Theorem and distance formula.
- I can complete the square of a quadratic equation.
- I can derive the equation of a circle using the Pythagorean Theorem – given coordinates of the center and length of the radius.
- I can determine the center and radius by completing the square.
- I can use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, 3) lies on the circle centered at the origin and containing the point (0,2).
- I can recall previous understandings of coordinate geometry (including, but not limited to: distance, midpoint and slope formula, equation of a line, definitions of parallel and perpendicular lines, etc.).

Instructional Strategies

Review the definition of a circle as a set of points whose distance from a fixed point is constant.

Review the algebraic method of completing the square and demonstrate it geometrically.

Illustrate conic sections geometrically as cross sections of a cone.

Use the Pythagorean theorem to derive the distance formula. Then, use the distance formula to derive the equation of a circle with a given center and radius, beginning with the case where the center is the origin.
Starting with any quadratic equation in two variables (x and y) in which the coefficients of the quadratic terms are equal, complete the squares in both x and y and obtain the equation of a circle in standard form.

Given two points, find the equation of the circle passing through one of the points and having the other as its center.

Define a parabola as a set of points satisfying the condition that their distance from a fixed point (focus) equals their distance from a fixed line (directrix). Start with a horizontal directrix and a focus on the y-axis, and use the distance formula to obtain an equation of the resulting parabola in terms of y and x^2. Next use a vertical directrix and a focus on the x-axis to obtain an equation of a parabola in terms of x and y^2. Make generalizations in which the focus may be any point, but the directrix is still either horizontal or vertical. Allow sufficient time for students to become familiar with new vocabulary and notation.

Given y as a quadratic equation of x (or x as a quadratic function of y), complete the square to obtain an equation of a parabola in standard form.

Common Misconceptions and Challenges

Because new vocabulary is being introduced in this cluster, remembering the names of the conic sections can be problematic for some students.

The Euclidean distance formula involves squared, subscripted variables whose differences are added. The notation and multiplicity of steps can be a serious stumbling block for some students.

The method of completing the square is a multi-step process that takes time to assimilate. A geometric demonstration of completing the square can be helpful in promoting conceptual understanding.

Performance Level Descriptors

Limited Recognize an equation of a circle (G.GPE.1)

Basic N/A

Proficient Write the equation of a circle given its center and radius (G.GPE.1)

Accelerated Use properties of circles to solve routine problems related to the equation of a circle, its center and radius length (G.GPE.1)

Complete the square to find the center and radius of a circle given by an equation (G.GPE.1)

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets Geometry

G.GPE.4	Essential Understanding	Academic Vocabulary/Language
Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special right triangles, quadrilaterals, and circles.For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.	Students can prove simple theorems. They will use coordinates of points to do this in the coordinate plane. **Extended Understanding** Students will be comfortable with irrational coordinates. Students will be able to approximate square roots and find them on a coordinate plane.	- coordinates - geometric theorem - origin - use - prove - disprove

Essential Skills

- I can recall previous understandings of coordinate geometry (including, but not limited to: distance, midpoint and slope formula, equations of a line, definitions of parallel and perpendicular lines, etc.).

- I can use coordinate to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, sqrt(3)) lies on the circle centered at the origin and containing the point (0,2).

Instructional Strategies

Illustrative Mathematics:

- G-GPE A Midpoint Miracle
- G-GPE, G-CO, G-SRT Unit Squares and Triangles

https://www.opened.com/homework/g-gpe-4-use-coordinates-to-prove-simple-geometric-theorems/3689764

Common Misconceptions and Challenges

Students often have trouble estimating square roots. Remind them to think of the nearest perfect square greater and less than the value they are given.

Performance Level Descriptors

Limited Given four points on the coordinate plane, determine if the points create a rectangle with horizontal and vertical sides

Basic N/A

Proficient Use coordinates to prove/disprove that a figure defined by four given points in the coordinate plane is a rectangle Use the slope criteria for parallel and perpendicular lines to solve geometric problems algebraically

Accelerated Use coordinates to prove/disprove that a given point lies on the circle centered on the origin and containing another given point

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets Geometry

G.GPE.5, G.GPE.7

5: Justify the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Essential Understanding

Students can prove simple theorems. They will use coordinates of points to do this in the coordinate plane.

Extended Understanding

Provide students opportunities to: find pictures of real world examples of parallel lines. They can use magazines, clip art, internet pictures or take pictures themselves. Overlaying graph paper on their picture, instruct them to prove the lines are parallel. ; use Google Earth to find a real-world shape (i.e, a metro park, their yard, the stadium at OSU). Ask the students to determine the perimeter and area of their diagram using coordinate geometry. Discuss scale factor with students, reminding them to use a realistic scale to determine the perimeter and area. (You might also have several students use the same picture so they can compare their perimeters and areas)

Academic Vocabulary/Language

coordinates
geometric theorem
origin
prove
disprove

Essential Skills

- I can recall previous understandings of coordinate geometry (including, but not limited to: distance, midpoint and slope formula, equations of a line, definitions of parallel and perpendicular lines, etc.).
- I can use coordinate to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, sqrt(3)) lies on the circle centered at the origin and containing the point(0,2).

Instructional Strategies

Review the concept of slope as the rate of change of the y-coordinate with respect to the x-coordinate for a point moving along a line, and derive the slope formula. Use similar triangles to show that every nonvertical line has a constant slope. Review the point-slope, slope-intercept and standard forms for equations of lines. Investigate pairs of lines that are known to be parallel or perpendicular to each other and discover that their slopes are either equal or have a product of –1, respectively. Pay special attention to the slope of a line and its applications in analyzing properties of lines. Allow adequate time for students to become familiar with slopes and equations of lines and methods of computing them. Use slopes and the Euclidean distance formula to solve problems about figures in the coordinate plane such as: Given three points, are they vertices of an isosceles, equilateral, or right triangle? Given four points, are they vertices of a parallelogram, a rectangle, a rhombus, or a square? Given the equation of a circle and a point, does the point lie outside, inside, or on the circle? Given the equation of a circle and a point on it, find an equation of the line tangent to the circle at that point. Given a line and a point not on it, find an equation of the line through the point that is parallel to the given line. Given a line and a point not on it, find an equation of the line through the point that is perpendicular to the given line. Given the equations of two non-parallel lines, find their point of intersection. Given two points, use the distance formula to find the coordinates of the point halfway between them. Generalize this for two arbitrary points to derive the midpoint formula. Use linear interpolation to generalize the midpoint formula and find the point that partitions a line segment in any specified ratio. Use the distance formula to find the length of each side of a polygon whose vertices are known, and compute the perimeter of that figure.

Common Misconceptions and Challenges

Students often have trouble estimating square roots. Remind them to think of the nearest perfect square greater and less than the value they are given.

Performance Level Descriptors

Limited Use slope criteria to determine if given lines are parallel or perpendicular (GPE.5)

Use coordinates to compute perimeters of polygons with sides that are horizontal or vertical (GPE.7)

Basic Use the slope criteria for parallel or perpendicular lines to solve geometric problems (GPE.5)

Use coordinates to compute areas of triangles and rectangles with horizontal and/or vertical sides (GPE.7)

Proficient Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g. distance formula) (GPE.7)

Accelerated N/A

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**Essential Understanding** Students will be able to find the point on a directed line segment. Students will able to determine a given ratio. **Extended Understanding** When given a ratio and a line segment with either a positive or negative slope, students will be able to find the point that partitions the segment into the given ratio.	**Academic Vocabulary/Language** - directed line segment - point - partitions - given ratio - recall - given - find

Essential Skills

- I can recall the definition of ratio.
- I can recall previous understandings of coordinate geometry.
- I can, given a line segment (including those with positive and negative slopes) and ratio, find the point on the segment that partitions the segment into the given ratio.

Instructional Strategies

This standard is not specifically covered in the textbook.

Show the 12 minute video at https://www.educreations.com/lesson/view/g-gpe-6-partitioning-a-segment/9730636/

Common Misconceptions and Challenges

When finding the length of a line segment in order to determine its midpoint, students may perform simple arithmetic incorrectly. For example, you are given the coordinates (4, -3) and (12, -8). Students sometimes want to remove the negative sign on the answer to -3 + -8.

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.SRT.1, G.SRT.2, G.SRT.3

1. Verify experimentally the properties of dilations given by a center and a scale factor.
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Essential Understanding

Students can use scale factor to determine how a pre-image and image relate to one another.

Extended Understanding

Students can determine if figures are similar or not by examining their sides and angle measures.

Academic Vocabulary/Language

- scale factor
- dilation
- center of dilation
- similarity
- transformation
- image / pre-image
- verify
- use the properties

Essential Skills

- I can define image, pre-image, scale factor, center, and similar figures as they relate to transformations.
- I can identify a dilation stating its scale factor and center.
- I can verify experimentally that a dilated image is similar to its pre-image by showing congruent corresponding angles and proportional sides.
- I can identify a dilation stating its scale factor and center.
- I can explain that the scale factor represents how many times longer or shorter a dilated line segment is than its pre-image.
- I can, by using similarity transformations, explain that triangles are similar if all pairs of corresponding angles are congruent and all corresponding pairs of sides are proportional.
- I can, given two figures, decide if they are similar by using the definition of similarity in terms of similarity transformations.
- I can recall the properties of similarity transformations.
- I can establish the AA criterion for similarity of triangles by extending the properties of similarity transformations to the general case of any two similar triangles.

Instructional Strategies

Allow adequate time and hands-on activities for students to explore dilations visually and physically.

Use graph paper and rulers or dynamic geometry software to obtain images of a given figure under dilations having specified centers and scale factors. Carefully observe the images of lines passing through the center of dilation and those not passing through the center, respectively. A line segment passing through the center of dilation will simply be shortened or elongated but will lie on the same line, while the dilation of a line segment that does not pass through the center will be parallel to the original segment (this is intended as a clarification of Standard 1a).

Illustrate two-dimensional dilations using scale drawings and photocopies.

Measure the corresponding angles and sides of the original figure and its image to verify that the corresponding angles are congruent and the corresponding sides are proportional (i.e. stretched or shrunk by the same scale factor). Investigate the SAS and SSS criteria for similar triangles.

Use graph paper and rulers or dynamic geometry software to obtain the image of a given figure under a combination of a dilation followed by a sequence of rigid motions (or rigid motions followed by dilation).

Work backwards – given two similar figures that are related by dilation, determine the center of dilation and scale factor. Given two similar figures that are related by a dilation followed by a sequence of rigid motions, determine the parameters of the dilation and rigid motions that will map one onto the other.

Using the theorem that the angle sum of a triangle is 180°, verify that the AA criterion is equivalent to the AAA criterion.

Given two triangles for which AA holds, use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that dilation will complete the mapping of one triangle onto the other.

Common Misconceptions and Challenges

Some students often do not recognize that congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes a congruency.

Students may not realize that similarities preserve shape, but not size. Angle measures stay the same, but side lengths change by a constant scale factor.

Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified.

Performance Level Descriptors

Limited N/A

Basic Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar (G.SRT.2)

Proficient Use the properties of similarity transformations to justify the AA-criterion for two triangles to be similar (G.SRT.3)

Accelerated Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides (G.SRT.2)

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.SRT.4, G.SRT.5

4. Prove and apply theorems both formally and informally about triangles using a variety of methods. Theorems include but are not restricted to: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

Essential Understanding

Students will continue to prove key elements of triangles.

Extended Understanding

Students will be able to solve problems and prove relationships in geometric figures by using congruence and similarity

Academic Vocabulary/Language

- theorem
- triangle
- parallel
- proportionally
- Pythagorean Theorem
- congruence
- similarity
- prove
- use
- solve problems

Essential Skills

- I can recall postulates, theorems, and definitions to prove theorems about triangles.
- I can prove theorems involving similarity about triangles. (Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity).

Instructional Strategies

Review triangle congruence criteria and similarity criteria, if it has already been established.
Review the angle sum theorem for triangles, the alternate interior angle theorem and its converse, and properties of parallelograms. Visualize it using dynamic geometry software.

Using SAS and the alternate interior angle theorem, prove that a line segment joining midpoints of two sides of a triangle is parallel to and half the length of the third side. Apply this theorem to a line segment that cuts two sides of a triangle proportionally.

Generalize this theorem to prove that the figure formed by joining consecutive midpoints of sides of an arbitrary quadrilateral is a parallelogram. (This result is known as the Midpoint Quadrilateral Theorem or Varignon's Theorem.)

Use cardboard cutouts to illustrate that the altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. Then use AA to prove this theorem. Then, use this result to establish the Pythagorean relationship among the sides of a right triangle ($a^2 + b^2 = c^2$) and thus obtain an algebraic proof of the Pythagorean Theorem.

Prove that the altitude to the hypotenuse of a right triangle is the geometric mean of the two segments into which its foot divides the hypotenuse.

Prove the converse of the Pythagorean Ttheorem, using the theorem itself as one step in the proof. Some students might engage in an exploration of Pythagorean Triples (e.g., 3-4-5, 5-12-13, etc.), which provides an algebraic

extension and an opportunity to explore patterns.

Common Misconceptions and Challenges

Some students may confuse the alternate interior angle theorem and its converse as well as the Pythagorean Theorem and its converse.

Performance Level Descriptors

Limited N/A

Basic Complete a straight-forward proof of a theorem involving proportionality of lengths within a triangle or among triangles by identifying one or two statements or reasons missing from the proof (G.SRT.4)

Proficient Use congruence and similarity criteria for triangles to solve routine problems (G.SRT.5)

Accelerated Use congruence and similarity criteria for triangles to prove relationships in geometric figures (G.SRT.5)

Use triangle similarity to construct a proof of a theorem involving proportionality of lengths within a triangle or among triangles (G.SRT.4)

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.SRT.6, G.SRT.7, G.SRT.8

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Solve problems involving right triangles.

   a. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given (excludes inverse trig functions)

Essential Understanding

Students will understand the foundation of trigonometric ratios for acute angles comes from the similarity between sides of right triangles.

Extended Understanding

There is a defined relationship between sine and cosine of complementary angles.

Academic Vocabulary/Language

- acute angle
- similarity
- trigonometric ratio
- sine
- cosine
- complementary angles
- understand
- explain and use
- solve
- applied problems

Essential Skills

- I can name the sides of right triangles as related to an acute angle.
- I can recognize that if two right triangles have a pair of acute, congruent angles that the triangles are similar.
- I can compare common ratios for similar right triangles and develop a relationship between the ratio and the acute angle leading to the trigonometry ratios.
- I can use the relationship between the sine and cosine of complementary angles.
- I can identify sine and cosine of acute angles in right triangles.
- I can recognize which methods could be used to solve right triangles in applied problems.
- I can solve for an unknown angle or side of a right triangle using sine, cosine, and tangent.

Instructional Strategies

Review vocabulary (opposite and adjacent sides, legs, hypotenuse and complementary angles) associated with right triangles.

Make cutouts or drawings of right triangles or manipulate them on a computer screen using dynamic geometry software and ask students to measure side lengths and compute side ratios. Observe that when triangles satisfy the AA criterion, corresponding side ratios are equal. Side ratios are given standard names, such as sine, cosine and tangent. Allow adequate time for students to discover trigonometric relationships and progress from concrete to abstract understanding of the trigonometric ratios.

Show students how to use the trigonometric function keys on a calculator. Also, show how to find the measure of an acute angle if the value of its trigonometric function is known.

Investigate sines and cosines of complementary angles, and guide students to discover that they are equal to one another. Point out to students that the "co" in cosine refers to the "sine of the complement."

Observe that, as the size of the acute angle increases, sines and tangents increase while cosines decrease.
Stress trigonometric terminology by the history of the word "sine" and the connection between the term "tangent" in trigonometry and tangents to circles.

Have students make their own diagrams showing a right triangle with labels showing the trigonometric ratios. Although students like mnemonics such as SOHCAHTOA, these are not a substitute for conceptual understanding. Some students may investigate the reciprocals of sine, cosine, and tangent to discover the other three trigonometric functions.

Use the Pythagorean theorem to obtain exact trigonometric ratios for 30°, 45°, and 60° angles.
Use cooperative learning in small groups for discovery activities and outdoor measurement projects.

Have students work on applied problems and project, such as measuring the height of the school building or a flagpole, using clinometers and the trigonometric functions.

Common Misconceptions and Challenges

Some students believe that right triangles must be oriented a particular way.

Some students do not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle.

Some students believe that the trigonometric ratios defined in this cluster apply to all triangles, but they are only defined for acute angles in right triangles.

Performance Level Descriptors

Limited N/A

Basic N/A

Proficient Explain the relationship between sine and cosine of complementary angles (G.SRT.7)

Use trigonometric ratios or the Pythagorean Theorem to solve routine real world problems involving right triangles (G.SRT.8)

Accelerated

Advanced Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in non-routine real world problems (G.SRT.8)

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.C.1, G.C.2

Understand and apply theorems about circles

1. Prove that all circles are similar using transformational arguments.
2. Identify and describe relationships among inscribed angles, radii, chords, tangents and arcs and use them to solve problems. Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Essential Understanding

Students should know that unlike polygons that have dimensions independent of one another (base and height, for instance), a circle's size depends only on one measurement: the radius r.

Students should know that since all aspects of a circle's size depend on r; the size can be changed of any circle simply by dilating the radius by a constant scale factor.

Extended Understanding

Provide opportunities for students to engage in activities that will allow them to enhance understanding such as:

ttp://learnzillion.com/lessonsets/427-prove-that-all-circles-are-similar

This is an all in one unit to prove all circles are similar. It includes talk about using translations and dilations as well as triangles to prove that all circles are similar.

Academic Vocabulary/Language

-center
-central angle
-centroid
-chord
-circle
-circumcenter
-ircumference
-circumscribed angle
-cyclic
-diameter
-dilations
-equidistant
-focus
-incenter
-inscribed angle
-proportions
-quadrilateral
-radian
-radius
-scaler
-similar
-translations

Essential Skills

- I can, using the fact that the ratio of diameter to circumference is the same for circles, prove that all circles are similar.
- I can, using definitions, properties, theorems, identiry and describe relationships among inscribed angles, radii, and chords. Include central, inscribed, and circumscribed angles.
- I can understand that inscribed angles on a diameter are right angles.
- I can understand that the radius of a circle is perpendicular to the tangent where the radius intersects the circle

Instructional Strategies

Given any two circles in a plane, show that they are related by dilation. Guide students to discover the center and scale factor of this dilation and make a conjecture about all dilations of circles. Starting with the special case of an angle inscribed in a semicircle, use the fact that the angle sum of a triangle is 180° to show that this angle is a right angle. Using dynamic geometry, students can grab a point on a circle and move it to see that the measure of the inscribed angle passing through the endpoints of a diameter is always 90°. Then extend the result to any inscribed angles. For inscribed angles, proofs can be based on the fact that the measure of an exterior angle of a triangle equals the sum of the measures of the nonadjacent angles. Consider cases of acute or obtuse inscribed angles. Use properties of congruent triangles and perpendicular lines to prove theorems about diameters, radii, chords, and tangent lines. Use formal geometric constructions to construct perpendicular bisectors of the sides and angle bisectors of a given triangle. Their intersections are the centers of the circumscribed and inscribed circles, respectively.

Common Misconceptions and Challenges

Students sometimes confuse inscribed angles and central angles. Students may think they can tell by inspection whether a line intersects a circle in exactly one point. It may be beneficial to formally define a tangent line as the line perpendicular to a radius at the point where the radius intersects the circle

Performance Level Descriptors

Limited Recognize definitions of angle, circle, perpendicular lines, parallel lines, and line segment (G.CO.1)

Identify central angles and find their measures given the measure of their intercepted arcs (G.CO.2)

Basic Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment (G.CO.1)

Identify inscribed angles and circumscribed angles (G.C.2)

Proficient Find the measure of inscribed angles and circumscribed angles given the measure of their intercepted arcs (G.C.2)

Use transformations between two or more circles to show similarity (G.CO.1)

Accelerated **N/A**

Advanced **N/A**

Ohio's Learning Standards – Clear Learning Targets Geometry

G.C.3, G.C.4

Understand and apply theorems about circles.

3. Construct the inscribed and circumscribed circles of a triangle; prove and apply the property of that opposite angles are supplementary for a quadrilateral inscribed in a circle,

4. Construct a tangent line from a point outside a given circle to the circle.

Essential Understanding

Students need to understand that when a circle is inscribed in a polygon, then the polygon is circumscribed about the circle and when a circle is circumscribed about a polygon, then the polygon is inscribed in the circle.

Students need to understand that when a circle is inscribed in a polygon, then the polygon is circumscribed about the circle and when a circle is circumscribed about a polygon, then the polygon is inscribed in the circle.

Extended Understanding

Challenge students to generalize the results about angle sums of triangles and quadrilaterals to a corresponding result for n-gons.

Students will construct a tangent to the circle

Academic Vocabulary/Language

-triangle
angles,
-centroid
-circumcenter
-circumscribe
-concurrent
-incenter
-inscribe
-inscribed arc
-inscribed angle
-inscribed quadrilateral
-orthocenter
-quadrilateral

Essential Skills

- I can construct inscribed circles of a triangle.
- I can construct circumscribed circles of a triangle.
- I can, using definitions, properties, and theorems, prove properties of angles for a quadrilateral inscribed in a circle

Instructional Strategies

Given any two circles in a plane, show that they are related by dilation. Guide students to discover the center and scale factor of this dilation and make a conjecture about all dilations of circles.

Starting with the special case of an angle inscribed in a semicircle, use the fact that the angle sum of a triangle is 180° to show that this angle is a right angle. Using dynamic geometry, students can grab a point on a circle and move it to see that the measure of the inscribed angle passing through the endpoints of a diameter is always 90°. Then extend the result to any inscribed angles. For inscribed angles, proofs can be based on the fact that the measure of an exterior angle of a triangle equals the sum of the measures of the nonadjacent angles. Consider cases of acute or obtuse inscribed angles.

Use properties of congruent triangles and perpendicular lines to prove theorems about diameters, radii, chords, and tangent lines. Use formal geometric constructions to construct perpendicular bisectors of the sides and angle bisectors of a given triangle. Their intersections are the centers of the circumscribed and inscribed circles, respectively.

Dissect an inscribed quadrilateral into triangles, and use theorems about triangles to prove properties of these quadrilaterals and their angles.

Common Misconceptions and Challenges

Students sometimes confuse inscribed angles and central angles.

Students may think they can tell by inspection whether a line intersects a circle in exactly one point. It may be beneficial to formally define a tangent line as the line perpendicular to a radius at the point where the radius intersects the circle.

Remembering which point of concurrency is created by the four special triangle segments. The medians make the centroid, the perpendicular bisectors make the circumcenter, the angle bisectors make the incenter, and the altitudes make the orthocenter.

Performance Level Descriptors

Limited N/A

Basic N/A

Proficient N/A

Accelerated Construct inscribed and circumscribed circles of a triangle (G.C.3)

Construct a proof of properties of angles for a quadrilateral inscribed in a circle (G.C.3)

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.C.5

Find arc lengths and areas of sectors of circles.

a. Apply similarirty to relate the length of an arc incepted by a central angle to the radius. Use the relationship to solve problems.
b. Derive the formula for the area of a sector and use it to solve problems.

Essential Understanding

Students are introduced to the concept of radian measure.
Students should recall that all circles are similar.

Extended Understanding

Students will derive the formula for the area of a sector.

Academic Vocabulary/Language

- similarity
- intercepted arc
- proportional
- radian measure
- constant of proportionality
- area of a sector
- derive
- define

Essential Skills

- I can recall how to find the area and circumference of a circle.
- I can explain that 1° = 180 radians.
- I can recall from G.C.1, that all circles are similar.
- I can determine the constant of proportionality.
- I can justify the radii of any two circles (r_1 and r_2) and the arc lengths (s_1 and s_2) determined by congruent central angles are proportional, such that $r_1 / s_1 = r_2 / s_2$.
- I can verify that the constant of a proportion is the same as the radian measure of the given central angle.

Instructional Strategies

Begin by calculating lengths of arcs that are simple fractional parts of a circle (e.g. 1/6), and do this for circles of various radii so that students discover a proportionality relationship.

Provide plenty of practice in assigning radian measure to angles that are simple fractional parts of a straight angle.

Stress the definition of radian by considering a central angle whose intercepted arc has its length equal to the radius, making the constant of proportionality 1. Students who are having difficulty understanding radians may benefit from constructing cardboard sectors whose angles are one radian. Use a ruler and string to approximate such an angle.

Compute areas of sectors by first considering them as fractional parts of a circle. Then, using proportionality, derive a formula for their area in terms of radius and central angle. Do this for angles that are measured both in degrees and radians and note that the formula is much simpler when the angels are measured in radians.

Derive formulas that relate degrees and radians.

Introduce arc measures that are equal to the) measures of the intercepted central angles in degrees or radians..

Emphasize appropriate use of terms, such as, angle, arc, radian, degree, and sector.

Common Misconceptions and Challenges

Sectors and segments are often used in everyday conversation. Care should be taken to distinguish these two geometric concepts.

The forumals for converting radians to degrees and vice versa are easily confused. Knowing that the degree measure of a given angle is always a number larger than the radian measure can heko students use the correct unit.

Performance Level Descriptors

Limited N/A

Basic N/A

Proficient N/A

Accelerated Use the formula for the area of a sector of a circle to solve routine problems

Advanced Solve non-routine real world problems involving finding arc lengths and areas of sectors

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.GMD.1, G.GMD.3

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Essential Understanding

Students will be able to explain the formula for the circumference of a circle.
Students will be able to explain Cavalieri's principle.

Extended Understanding

Students will be able to use volume formulas to solve problems

Academic Vocabulary/Language

- informal argument
- circumference
- volume
- cylinder
- cone
- dissection argument
- Cavalieri's principle

Essential Skills

- I can recognize cross-sections of solids as two-dimensional shapes.
- I can recognize formulas for area and circumference of a circle and volume of a cylinder, pyramid, and cone.
- I can use the techniques of dissection and limit arguments.
- I can recognize Cavalieri's principle.
- I can decompose volume formulas into area formulas using cross-sections.
- I can apply dissection and limit arguments (e.g. Archimedes' inscription and circumscription of polygons about a circle) and as a component of the informal argument for the formulas for the circumference and area of a circle.
- I can apply Cavalieri's Principle as a component of the informal argument for the formulas for the volume of a cylinder, pyramid, and cone.
- I can utilize the appropriate formula for volume depending on the figure.
- I can use volume formulas for cylinders, pyramids, cones, and spheres to solve contextual problems.

Instructional Strategies

Revisit formulas $C = \pi d$ and $C = 2\pi r$. . Observe that the circumference is a little more than three times the diameter of the circle. Briefly discuss the history of this number and attempts to compute its value.

Use alternative ways to derive the formula for the area of the circle $A = \pi r^2$. For example,
Cut a cardboard circular disk into 6 congruent sectors and rearrange the pieces to form a shape that looks like a parallelogram with two scalloped edges. Repeat the process with 12 sectors and note how the edges of the parallelogram look "straighter." Discuss what would happen in the case as the number of sectors becomes infinitely large. Then calculate the area of a parallelogram with base ½ *C* and altitude *r* to derive the formula $A = \pi r^2$.

Wind a piece of string or rope to form a circular disk and cut it along a radial line. Stack the pieces to form a triangular shape with base *C* and altitude *r*. Again discuss what would happen if the string became thinner and thinner so that the number of pieces in the stack became infinitely large. Then calculate the area of the triangle to derive the formula $A = \pi r^2$.

Introduce Cavalieri's principle using a concrete model, such as a deck of cards. Use Cavalieri's principle with cross sections of cylinders, pyramids, and cones to justify their volume formulas.

For pyramids and cones, the factor $\frac{1}{3}$ will need some explanation. An informal demonstration can be done using a volume relationship set of plastic shapes that permit one to pour liquid or sand from one shape into another. Another way to do this for pyramids is with Geoblocks®. The set includes three pyramids with equal bases and altitudes that will stack to form a cube. An algebraic approach involves the formula for the sum of squares ($1^2 + 2^2 + \ldots + n^2$).

After the coefficient 1/3 has been justified for the formula of the volume of the pyramid ($A = \frac{1}{3}Bh$), one can argue that it must also apply to the formula of the volume of the cone by considering a cone to be a pyra*Type equation here.*mid that has a base with infinitely many sides.

The formulas for volumes of cylinders, pyramids, cones and spheres can be applied to a wide variety of problems such as finding the capacity of a pipeline; comparing the amount of food in cans of various shapes; comparing capacities of cylindrical, conical and spherical storage tanks; using pyramids and cones in architecture; etc. Use a combination of concrete models and formal reasoning to develop conceptual understanding of the volume formulas.

Some information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics, and goes beyond the mathematics that all students should study in order to be college- and career-ready:

Common Misconceptions and Challenges

An informal survey of students from elementary school through college showed the number pi to be the mathematical idea about which more students were curious than any other. There are at least three facets to this curiosity: the symbol π itself, the number 3.14159…, and the formula for the area of a circle. All of these facets can be addressed here, at least briefly.

Many students want to think of infinity as a number. Avoid this by talking about a quantity that becomes larger and larger with no upper bound.

The inclusion of the coefficient $\frac{1}{3}$ in the formulas for the volume of a pyramid or cone and $\frac{4}{3}$in the formula for the volume of a sphere remains a mystery for many students. In high school, students should attain a conceptual understanding of where these coefficient come from. Concrete demonstrations, such as pouring water from one shape into another should be followed by more formal reasoning.

Performance Level Descriptors

Limited Use volume formulas to find volumes of cylinders, pyramids, cones, and spheres, given all needed measurements, to solve simple problems (G.MD.3)

Basic N/A

Proficient Use volume formulas involving finding a measurement (e.g. height or radius) of cylinders, pyramids, cones, and spheres, given the volume and other measurements, to solve problems (G.MD.3)

Accelerated N/A

Advanced Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone (G.MD.1)

Ohio's Learning Standards – Clear Learning Targets Geometry

G.GMD.4, G.GMD.5, G.GMD.6

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

5. Understand how and when changes to the measures of a figure (lengths and angles) result in similar and non-similar figures.

6. When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on the lengths, areas, and volumes are multiplied by k, k^2, and k^3, respectively.

Essential Understanding

When given a three-dimensional object, students will be expected to identify the shape made when the object is cut into cross sections.

Students are expected to know the three-dimensional figure that is generated when a two dimensional figure is rotating.

Students are expected to know that a cross section of a solid is an intersection of a plane (two dimensional) and a solid (three-dimensional).

Extended Understanding

Provide opportunities such as the following, for students to engage in experiences using skills learned in this sections:

Tennis Balls in a Can

http://www.illustrativemathematics.org/illustrations/512

a real life situation using a can of tennis balls and an x-ray machine at the airport to see the cross sections of the can, and to determine what the cross section would look like in different circumstances.

Academic Vocabulary/Language

-area
-base
-bisect
-circle
-circumference
-construct
-coplanar
-cone
-cross section
-cutting plane
-cube
-cylinder
-diameter
-dimension
-equilateral
-line
-parallel
-perpendicular
-pi
-plane
-radius
-regular
-rotation
-slid
-solid of revolution
-volume

Essential Skills

- I I can, given a three- dimensional object, identify the shape made when the object is cut into cross-sections.
- I can, when rotating a two- dimensional figure, such as a square, know the three-dimensional figure that is generated, such as a cylinder. Understand that a cross section of a solid is an intersection of a plane (twodimensional) and a solid (three-dimensional).

Instructional Strategies

Review vocabulary for names of solids (e.g., right prism, cylinder, cone, sphere, etc.). Slice various solids to illustrate their cross sections. For example, cross sections of a cube can be triangles, quadrilaterals or hexagons. Rubber bands may also be stretched around a solid to show a cross section. Cut a half-inch slit in the end of a drinking straw, and insert a cardboard cutout shape. Rotate the straw and observe the three-dimensional solid of revolution generated by the two-dimensional cutout. Java applets on some web sites can also be used to illustrate cross sections or solids of revolution. Encourage students to create three-dimensional models to be sliced and cardboard cutouts to be rotated. Students can also make three-dimensional models out of modeling clay and slice through them with a plastic knife.

Common Misconceptions and Challenges

Some cross sections are more difficult to visualize than others. For example, it is often easier to visualize a rectangular cross section of a cube than a hexagonal cross section. Generating solids of revolution involves motion and is difficult to visualize by merely looking at drawings.

Performance Level Descriptors

Limited N/A

Basic Identify the shapes of two-dimensional cross-sections of three-dimensional objects

Proficient Identify three-dimensional objects generated by rotations of two-dimensional objects

Accelerated N/A

Advanced N/A

Ohio's Learning Standards – Clear Learning Targets
Geometry

G.MG.1, G.MG.2, G.MG.3

Apply geometric concepts in modeling situations in modeling.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize

Essential Understanding

Students are expected to apply and model geometric concepts.

Extended Understanding

Encourage students to engage in a project(s) using real-world applications of geometry. Resources: A Sourcebook of Applications of School Mathematics, compiled by a Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics (1980); Mathematics: Modeling our World, Course 1 and Course 2, by the Consortium for Mathematics and its Applications (COMAP); Geometry & its Applications (GeoMAP) -- an exciting National Science Foundation project to introduce new discoveries and real-world applications of geometry to high school students. Produced by COMAP; Measurement in School Mathematics, NCTM 1976 Yearbook.

Academic Vocabulary/Language

-gometric concepts
-geometric methods
-properties

-analyze
-describe
-design
-model
-solve

Essential Skills

- I can use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- I can use the concept of density when referring to situations involving area and volume models, such as persons per square mile.
- I can solve design problems by designing an object or structure that satisfies certain constraints, such as minimizing cost or working with a grid system based on ratios (i.e., The enlargement of a picture using a grid and ratios and proportions)

Instructional Strategies

Genuine mathematical modeling typically involves more than one conceptual category. For example, modeling a herd of wild animals may involve geometry, measurement, proportional reasoning, estimation, probability and statistics, functions, and algebra. It would be somewhat misleading to try to teach a unit with the title of "modeling with geometry." Instead, these standards can be woven into other content clusters. A challenge for teaching modeling is finding problems that are interesting and relevant to high school students and, at the same time, solvable with the mathematical tools at the students' disposal. The resources listed below are a beginning for addressing this difficulty.

Common Misconceptions and Challenges

When students ask to see "useful" mathematics, what they often mean is, "Show me how to use this mathematical concept or skill to solve the homework problems." Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used.

Performance Level Descriptors

Limited N/A

Basic N/A

Proficient Use the measures of geometric shapes and their properties to describe real-world objects (G.MG.1)

Apply concepts of density based on volume in modeling situations (G.MG.2)

Use geometric methods to solve routine design problems limited by constraints or restrictions (G.MG.3)

Accelerated Use geometric methods to model design problems limited by constraints or restrictions (G.MG.3)

Advanced Use a variety of geometric methods to model and solve non-routine design problems limited by many constraints or restrictions (G.MG.3)

Ohio's Learning Standards – Clear Learning Targets
Geometry

S.CP.1, S.CP.2, S.CP.3, S.CP.4, S.CP.5

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Essential Understanding

Students will multiply probabilities to find the probability of independent events.

Students will know the difference between dependent and independent events. They will be able to determine if the outcome of one event has an impact on the outcome of another event.

Extended Understanding

Students will be able to find real world situations that model conditional probability and independence.

Academic Vocabulary/Language

- union
- intersection
- subset
- event
- independence
- sample space
- probability
- recognize
- explain
- describe
- construct
- interpret

Essential Skills

- I can define unions, intersections and complements of events.
- I can describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events.
- I can determine the outcome of independent events as the product of their probabilities.
- I can categorize events as independent or not using the characterization that two events A and B are independent when the probability of A and B occurring together is the product of their probabilities.
- I can recognize the conditional probability of A given B is the same as P(A and B)P(B).
- I can interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as probability of B.
- I can use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- I can recognize the concepts of conditional probability and independence in everyday language and everyday situations.

Instructional Strategies

The **Standard for Mathematical Practice**, *precision* is important for working with conditional probability. Attention to the definition of an event along with the writing and use of probability function notation are important requisites for communication of that precision. For example: Let A: Female and B: Survivor, then P(A|B) =. The use of a vertical line for the conditional "given" is not intuitive for students and they often confuse the events B|A and A|B. Moreover, they often find identifying a conditional difficult when the problem is expressed in words in which the word "given" is omitted. For example, find the probability that a female is a survivor. The standard ***Make sense of problems and persevere in solving them*** also should be employed so students can look for ways to construct conditional probability by formulating their own questions and working through them such as is suggested in standard 4 above. Students should learn to employ the use of Venn diagrams as a means of finding an entry into a solution to a conditional probability problem.

It will take a lot of practice to master the vocabulary of "or," "and," "not" with the mathematical notation of union (∪), intersection (∩), and whatever notation is used for complement.

The independence of two events is defined in Standard 2 using the intersection. It is far more intuitive to introduce the independence of two events in terms of conditional probability (stated in Standard 3), especially where calculations can be performed in two-way tables.

The Standards in this cluster deliberately do not mention the use of tree diagrams, the traditional way to treat conditional probabilities. Instead, probabilities of conditional events are to be found using a two-way table wherever possible. However, tree diagrams may be a helpful tool for some students. The difficulty is realizing that the second set of branches are conditional probabilities.

Common Misconceptions and Challenges

Students may believe that multiplying across branches of a tree diagram has nothing to do with conditional probability.
Students may believe that "independence of events" and "mutually exclusive events" are the same thing.

Performance Level Descriptors

Limited Identify the sample space (the set of outcomes) using characteristics of the outcomes (S.CP.1)

Complete two-way frequency tables of data when two categories are associated with each object being classified (S.CP.4)

Basic Describe events as subsets of a sample space (the set of outcomes) using categories of the outcomes (S.CP.1)

Use two-way frequency tables of data when two categories are associated with each object being classified as a sample space to determine probabilities(S.CP.4)

Recognize independence of events in everyday language and everyday situations (S.CP.5)

Proficient Describe events as unions, intersections, or complements of other events using the terminology "or," "and," "not" (S.CP.1)

Determine the independence of two events in terms of the product of their probabilities (S.CP.2)

Understand the conditional probability of A given B as P(A and B)/P(B) (S.CP.3)

Determine and interpret independence of two events using products of probabilities, conditional probabilities, and two-way frequency tables (S.CP.4)

Recognize conditional probability in everyday language and everyday situations (S.CP.5)

Apply the addition rule for probability for events that are not mutually exclusive (S.CP.7)

Accelerated Describe events involving unions, intersections, or complements of other events using set notation (S.CP.1)

Use a two-way relative frequency table as a sample space to decide if events are independent by approximating conditional probabilities (S.CP.4)

Explain the independence of events in everyday situations (S.CP.5)

Advanced Explain the concepts of conditional probability in everyday situations (S.CP.5)

Apply the addition rule for probability for events that are not mutually exclusive and interpret the answer in terms of the model (S.CP.7)

Ohio's Learning Standards – Clear Learning Targets
Geometry

S.CP.6, S.CP.7

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Essential Understanding

Students will know how to find conditional probabilities.

Students will be able to interpret answers found in models.

Extended Understanding

Students will become familiar with the Addition Rule and Multiplication Rule.

Academic Vocabulary/Language

- permutation
- combination
- multiplication rule
- addition rule
- find
- apply
- use

Essential Skills

- I can explain the concepts of conditional probability and independence in everyday language and everyday situations.
- I can find the conditional probability of A given B as the fraction of B's outcomes that also belong to A.
- I can use the Additional Rule, P(A or B) = P(A)+P(B)–P(A and B).
- I can interpret the answer in terms of the model.
- I can use the multiplication rule with correct notation
- I can apply the general Multiplication Rule in a uniform probability model P(A and B) = P(A)P(B|A)] = P(B)P(A|B)
- I can interpret the answer in terms of the model.
- I can identify situations that are permutations and those that are combinations.
- I can use permutations and combinations to compute probabilities of compound events and solve problems.

Instructional Strategies

Identifying that a probability is conditional when the word "given" is not stated can be very difficult for students. For example, if a balanced tetrahedron with faces 1, 2, 3, 4 is rolled twice, what is the probability that the sum is prime (A) of those that show a 3 on at least one roll (B)? Whether what is asked for is P(A and B), P(A or B), or P(A|B) can be problematic for students. Showing the outcomes in a Venn Diagram may be useful. The calculation to find the probability that the sum is prime (A) given at least one roll shows 3 (B) is to count the elements of B by listing them if possible, namely in this example, there are 7 paired outcomes (31, 32, 33, 34, 13, 23, 43). Of those 7 there are 4 whose sum is prime (32, 34, 23, 43). Hence in the long run, 4 out of 7 times of rolling a fair tetrahedron twice, the sum of the two rolls will be a prime number under the condition that at least one of its rolls shows the digit 3.

Note that if listing outcomes is not possible, then counting the outcomes may require a computation technique involving permutations or combinations, which is a STEM topic.

In the above example, if the question asked were what is the probability that the sum of two rolls of a fair tetrahedron is prime (A) or at least one of the rolls is a 3 (B), then what is being asked for is P(A or B) which is denoted as $P(A \cup B)$ in set notation. Again, it is often useful to appeal to a Venn Diagram in which A consists of the pairs: 11, 12, 14, 21, 23, 32, 34, 41, 43; and B consist of 13, 23, 33, 43, 31, 32, 34. Adding P(A) and P(B) is a problem as there are duplicates in the two events, namely 23, 32, 34, and 43. So P(A or B) is 9/16 + 7/16 – 4/16 = 12/16 or 3/4, so 3/4th of the time, the result of rolling a fair tetrahedron twice will result in the sum being prime, or at least one of the rolls showing a 3, or perhaps both will occur.

It should be noted that the Multiplication Rule in Standard 8 is designated as STEM when it is connected to the discussion of independence in Standard 2 of the previous S-CP cluster. The formula P(A and B) = P(A)P(B|A) is best illustrated in a two-stage setting in which A denotes the outcome of the first stage, and B, the second. For example, suppose a jar contains 7 red and 3 green chips. If one draws two chips without replacement from the jar, the probability of getting a red followed by a green is P(red on first and green on second) = P(red on first)P(green on second given a red on first) = (7/10)(3/9) = 21/90. Demonstrated on a tree diagram indicates that the conditional probabilities are on the second set of branches.

Common Misconceptions and Challenges

Students may believe that the probability of A or B is always the sum of the two events individually.

Students may believe that the probability of A and B is the product of the two events individually, not realizing that one of the probabilities may be conditional.

Students often switch permutations and combinations. Remind them that the order of their locker combination matters; however, it's the opposite in math: when the order matters, it's a permutation. Perhaps your locker should have a permutation, instead of a combination.

Performance Level Descriptors

Limited N/A

Basic Apply the addition rule for probability for events that are mutually exclusive (S.CP.7)

Proficient Apply the addition rule for probability for events that are not mutually exclusive (S.CP.7)

Accelerated N/A

Advanced Apply the addition rule for probability for events that are not mutually exclusive and interpret the answer (S.CP.7)