7th Grade Math

Pacing Guide and Unpacked Standards

GROVEPORT MADISON SCHOOLS

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Groveport Madison Math Pacing Guide – Grade 7

➤ Indicates Blueprint Focus Standards

7th	Ratios & Proportional Relationships	The Number System	Expressions & Equations	Geometry	Statistics & Probability	Standards for Mathematical Practice
1st 9 Weeks	➤**7.RP.1** Compute & analyze proportional relationships & use them to solve real-world & mathematical problems ➤**7.RP.2 (a,b,c,d)** Recognize & analyze proportional relationships & use them to solve real-world & mathematical problems	➤**7.NS.1 (a,b,c,d)** apply & extend understanding operations of fraction addition & subtraction; multiply & divide rational numbers ➤**7.NS.2 (a,b,c)** apply & extend understanding of operations w/fractions to add, subtract & divide rational numbers ➤**7.NS.3** apply & extend understanding of fractions to add, subtract, multiply, & divide rational numbers & manipulating fractions to complex fractions				**MP.1** Make sense of problems and persevere in solving them **MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the reasoning of others **MP.4** Model with mathematics **MP.5** Use appropriate tools strategically **MP.6** Attend to precision **MP.7** Look for and make use of structure **MP.8** Look for and express regularity in repeated reasoning
2nd 9 Weeks	➤**7.RP.3** Use proportional relationships to solve real-world & mathematical problems	➤**7.NS.3** apply & extend understanding of of fractions to add, subtract, multiply, & divide rational numbers & manipulating fractions to complex fractions	➤**7.EE.1** Apply properties of operations to generate equivalent expressions ➤**7.EE.2** Understand how writing equivalent expressions reveals properties of quantities ➤**7.EE.3** Solve- multi step real-life & mathematical problems ➤**7.EE.4 (a,b)** Use variables to represent values in real-world & algebraic expressions			

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➤ Indicates Blueprint Focus Standards

7th	Ratios & Proportional Relationships	The Number System	Expressions & Equations	Geometry	Statistics & Probability	Standards for Mathematical Practice
3rd 9 Weeks				➤**7.G.1 (a,b)** solve problems involving similar figures with triangles and quadrilaterals ➤**7.G.2 (a,b)** Draw, construct & manipulate quadrilaterals & describe their relationships w/ other geometric figures ➤**7.G.3** describe 2D figures that come from deconstructing 3D figures ➤**7.G.4 (a,b)** Explore, understand, & know components of circles & how to use them to solve real-world & mathematical problems ➤**7.G.5** Use facts about angles in multi-step problems to write & solve simple equations for unknown angles ➤**7.G.6** Solve real-world & mathematical problems involving area, volume, & surface of 2D / 3D figures		**MP.1** Make sense of problems and persevere in solving them **MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the reasoning of others **MP.4** Model with mathematics **MP.5** Use appropriate tools strategically **MP.6** Attend to precision **MP.7** Look for and make use of structure **MP.8** Look for and express regularity in repeated reasoning
4th 9 Weeks					➤**7.SP.1 (a,b,)** Understand that statistics can be used to explain information about a population through a sample ➤**7.SP.2 (a,b,c,d)** Broaden statistical reasoning by using the GAISE mode ➤**7.SP.3 (a,b)** Describe and analyze distributions ➤**7.SP.5** Understand probability and chance ➤**7.SP.7 (a,b)** Develop a probability model & use it to find probabilities of events ➤**7.SP.8 (a,b,c)** Find probabilities of compound events using organized lists, tables, tree diagrams, & simulation	

7.NS.1-3

Ohio's Learning Standards – Clear Learning Targets Math 7

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to 0.
b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real- world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, p -•- q = p + (-•-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-•-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-•-1)(-•-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-•-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non- zero divisor) is a rational number. If p and q are integers, then -•-(p/q) = (-•-p)/q = p/(-•-q). Interpret quotients of rational numbers by describing real-•-world contexts.
c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with multiple rational numbers extend the rules for manipulating fractions to complex fractions.

Essential Understanding

Two integers that are opposites are called additive inverses. Their sum is zero.

Numbers that are the same distance from zero on a number line have the same absolute value.

Specific rules for operations with integers are different from rules for operations with positive numbers alone.

To subtract an integer, add its additive inverse.

Convert a rational number to a decimal by using long division.

A rational number can be expressed as a ratio of two integers written as a fraction in which the denominator is not zero.

Vocabulary

- addition terms: sum, addend, more, etc.
- subtraction terms: difference, less, below, etc.
- multiplication terms: factor, product, of, etc.
- division terms: quotient, per, by, equal shares, etc.
- whole numbers
- counting numbers
- natural numbers
- integers
- rational number
- repeating decimal
- terminating decimal
- complex fraction
- opposites
- zero pairs
- absolute value
- additive inverse
- convert
- solve
- simplify

Essential Skills

- I can evaluate expressions using the order of operations (including using parenthesis, brackets, or braces).
- I can identify rules of addition and subtraction when adding and subtracting rational numbers.
- I can apply rules of operations as strategies to add and subtract rational numbers.
- I can describe situations in which opposite quantities combine to make 0.
- I can represent and explain how a number and its opposite have a sum of 0 and are additive inverses.
- I can convert subtraction of rational numbers to adding by using the additive inverse property, $p-q= p+(-q)$.
- I can apply rules of operations as strategies to multiply and divide rational numbers.
- I can convert a rational number to a decimal using long division, except when the divisor is zero.
- I can solve real-world mathematical problems by adding, subtracting, multiplying, and dividing rational numbers, including complex fractions.
- I can assess the reasonableness of answers using mental computation and estimation strategies

Instructional Strategies

This cluster builds upon the understandings of rational numbers in Grade 6:

- quantities can be shown using positive/negative symbols, or as having opposite directions/values,
- points on a number line show distance and direction,
- positive/negative numbers indicate locations on opposite sides of 0 on the number line,
- the opposite of an opposite is the number itself,
- the absolute value of a rational number is its distance from 0 on the number line,
- the absolute value is the magnitude for a positive or negative quantity, and
- locating and comparing locations on a coordinate grid by using negative and positive numbers.

Learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers.

Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Number lines present a visual image for students to explore and record addition and subtraction results.

Two-color counters or colored chips can be used as a physical and kinesthetic model for adding and subtracting integers. With one color designated to represent positives and a secondcolor for negatives. Addition/subtraction can be represented by placing the appropriate numbers of chips, for the addends and their signs, on a board. Using the notion of opposites, the board is simplified by removing pairs of opposite colored chips (zero pairs). The answer is the total of the remaining chips with the sign representing the appropriate color. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules.

Fractional numbers and whole numbers should be used in computations and explorations. Students should be able to give contextual examples of integer operations, write and solve equations for real-world problems and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers.

Table 1	Table 2	Table 3
4 x 4 = 16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x -4 =
4 x -2 =	-4 x 2 =	-2 x -4 =
4 x -3 =	-4 x 3 =	-3 x -4 =
4 x -4 =	-4 x 4 =	-4 x -4 =

Using the language of "the opposite of" helps some students understand the multiplication of negative numbers (-4 x -4 = 6), the opposite of 4 groups of -4. Discussion about the tables should address the patterns in theproducts, the role of the signs in the products and how the order of factors doesn't affect the product because of the commutative property. Then students should be asked to answer these questions and prove their responses.

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- How is the numerical value of the product of anytwo numbers found?
- When three factors are multiplied, how is the sign of the product determined?

Common Misconceptions and Challenges

Students get the term “opposite” and the term “inverse” mixed up. While most teachers realize what a student means when he says “Division is the opposite of multiplication”, encourage the student to use the correct term: inverse. The term “opposite” refers to the sign (and direction) of a number. The term “inverse” applies to operations such as addition and subtraction.

When applying the rules for adding a positive and a negative integer, students will subtract the “smaller” number from the “bigger” one. Encourage students to say the “number with the greatest or least absolute value” instead so that they are practicing and using vocabulary correctly.

Students must differentiate between the rules for addition of integers versus the rules for multiplication/division of integers. Students will commonly misapply the rules for multiplication/division to the operation of addition of integers.

Criteria for Success (Performance Level Descriptors)

- **Limited:** Model addition and subtraction of simple rational numbers on the number line; Recognize the additive inverse property
- **Basic:** Add, subtract, multiply, and divide integers; Convert between familiar fractions and decimals
- **Proficient:** Solve mathematical problems using the four operations on simple rational numbers; Convert from fractions to decimals without technology
- **Accelerated**: Solve mathematical problems using the four operations on rational numbers
- **Advanced:** Solve mathematical problems using the four operations on rational numbers

Prior Knowledge

6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

6.NS.2 Fluently divide multi-digit numbers using a standard algorithm.

6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.

Future Learning

8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is nonrepeating and non-terminating.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2.

8.NS.3 – N/A

Career Connections

Professions that utilize these skills

Accountant, Auditor, Claim Adjustor, Engineers (Aerospace Engineer, Biomedical Engineer, Chemical Engineer, Civil Engineer, Computer Hardware Engineer)

7.RP.1-3

Ohio's Learning Standards – Clear Learning Targets Math 7

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7.**RP.2** Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Essential Understanding

Students will use ratios or rates to compare two quantities.

Unit rates will be solved as fractions with a denominator of one.

Students will be able to solve a complex fraction by restructuring the problem as a fraction divided by a fraction.

Students will analyze tables and graphs to determine if proportionality exists.

Students will create proportions to solve for missing information.

Students will be able to describe relationships between quantities and/or proportionality in words.

Students will identify parts of a coordinate plane.

Students will be able to determine slope of a line through tables or graphs.

Students will determine percent of a number. (tax, tip, markdown, etc.)

Students will determine percent *of* increase/decrease and error.

Vocabulary

- ratio
- equivalent ratio
- rate
- unit rate
- complex fraction
- proportional relationship
- nonproportional relationship
- constant
- proportion
- cross products
- coordinate plane
- origin
- quadrants
- X and Y axis
- X and Y coordinate
- ordered pair
- slope
- X and Y intercept
- percent
- discount
- gratuity/tip
- markup
- markdown
- simple interest
- sales tax
- percent of increase/decrease
- percent error
- solve
- simplify

Essential Skills

- I can compute unit rates.
- I can analyze two ratios to determine proportionality using a variety of strategies (e.g., using tables, graphs, pictures, etc.).
- I can analyze tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships to identify the constant of proportionality.
- I know that a proportion is a statement of equality between two ratios.
- I can recognize what (0, 0) represents on the graph of a proportional relationship. Recognize what (1, r) on a graph represents, where r is the unit rate.
- I can explain what the points on a graph of a proportional relationship mean in terms of a specific situation.
- I can represent proportional relationships by writing equations.
- I can apply proportional reasoning to solve multi-step ratio and percent problems.
- I can assess the reasonableness of answers using mental computation and estimation strategies

Instructional Strategies

Building from the development of rate and unit rate concepts in Grade 6, applications now need to focus on solving unit-•-rate problems with more sophisticated numbers: fractions per fractions

Proportional relationships are further developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. Students are encouraged to fully understand the importance of a straight line through the origin on a graph, which denotes a proportional relationship. This understanding should precede the use of proportions and using cross products to find a missing value.

Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on "out of 100"; now percents above 100 are encountered.

Providing opportunities to solve problems based within contexts that are relevant to seventh graders will connect meaning to rates, ratios and proportions. Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-•- ended problem scenarios with and without numbers to give students the opportunity to demonstrate conceptual understanding, inviting students to create a similar problem to a given problem and explain their reasoning.

Common Misconceptions and Challenges

Sometimes students forget that the x-•-axis runs left to right and the y-•-axis goes up and down. The coordinates of a point are in alphabetical order: x, then y.

Also, make sure students know horizontal is left to right, while vertical is up and down.

Common mistakes involving percentages occur with percentages less than 10%. For instance, make sure students know the difference between 5% and 50%.

Criteria for Success (Performance Level Descriptors)

- **Limited**: Compute a unit rate of two whole numbers where the unit rate is explicitly requested; Identify proportional relationships presented in familiar contexts; Solve a one-step, straightforward ratio or percent problem
- **Basic:** Compute a unit rate of two familiar rational numbers where the unit rate is explicitly requested; Find the whole number constant of proportionality in relationships presented in familiar contexts; Solve a one-step, straightforward real-world ratio or percent problem
- **Proficient:** Compute a unit rate of two rational numbers where the unit rate is not explicitly requested; Represent proportional relationships in various formats; Use proportional relationships to solve routine real-world and mathematical ratio and percent problems with multiple steps
- **Accelerated:** Compare unit rates in a real-world context; Use different representations of promotional relationships to solve real-world problems; Apply proportional relationships to routine real-world and mathematical ratio and percent problems with multiple steps
- **Advanced:** Analyze and graph of a proportional relationship in order to explain what the points (x,y) and (1,r) represent, where r is the unit rate, and use it to solve problems; Apply proportional relationships to non-routine real-world and mathematical ratio and percent problems with multiple steps; Interpret products and quotients of rational numbers in real-world contexts

Prior Knowledge	Future Learning:
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. **6.RP.2** Understand the concept of a unit rate a /b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship **6.RP. 3**.Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	**8.F.1** Students will understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Career Connections

Professions that utilize these skills

Accountant, Auditor, Claim Adjustor, Engineers (Aerospace Engineer, Biomedical Engineer, Chemical Engineer, Civil Engineer, Computer Hardware Engineer), Stock Clerk

7.EE.1-4

Ohio's Learning Standards – Clear Learning Targets Math 7

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p – 0.15p) is equivalent to (1-.15)p, which is equivalent to 0.85p or finding 85% of the original price*

7.EE.3 Solve multi-step, real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies**.**

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

For example, a + 0.05a = 1.05a means that "Increase by 5%" is the same thing as "Multiply by 1.05".

Essential Understanding

Students will identify and utilize number properties in order to simplify expressions, equations and inequalities.

Students will translate numeric or algebraic expressions to words and vice versa.

Students will identify and combine like terms. Like terms contain the same variables raised to the same power; only the coefficients are different.

Students will write and solve equations.

Students will write, solve and graph inequalities.

Vocabulary

- associative property
- commutative property
- identity property
- distributive property
- factor
- expression
- equation
- inequality
- term
- coefficient
- variable
- constant
- like terms
- isolate the variable
- inverse operations

- addition terms: sum, addend, more, etc.
- subtraction terms: difference, less, below, etc.
- multiplication terms: factor, product, of, etc.
- division terms: quotient, per, by, equal shares, etc.

Essential Skills

- I can apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- I can factor and expand linear expressions with rational coefficients using the distributive property.
- I can write equivalent expressions with fractions, decimals, percents, and integers.
- I can apply properties of operations to calculate with numbers in any form
- I can identify and combine like terms. Examples: 3a + 4a = 7a, 2ab - 2a + 4ab = 6ab - 2a, 4ab + 2a CANNOT BE COMBINED.
- I can compare an algebraic solution to an arithmetic solution by identifying the sequence of the operations used in each approach.
- I can solve equations by isolating the variable and using inverse operations.
- I understand that in order to solve equations correctly, I must do exactly the same to each side of the equation to maintain its equivalency.
- I can use variables and construct equations to represent quantities of the form $px + q = r$ and $p(x + q) = r$ from real-world and mathematical problems
- I can solve inequalities using the same process as for equations with one exception: when multiplying or dividing by a negative, the inequality sign is switched to its opposite. Example: $\frac{-4v}{-4} > \frac{8}{-4}$

  $v < -2$

- I can graph an inequality on a number line using open and closed circles appropriately.
- I can solve and graph the solution set of the inequality of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers
- I can assess the reasonableness of answers using mental computation and estimation strategies

Instructional Strategies

Have students build on their understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions that were developed in Grade 6. Students continue to use properties that were initially used with whole numbers and now develop the understanding that properties hold for integers, rational and real numbers. Provide opportunities to build upon this experience of writing expressions using variables to represent situations and use the properties of operations to generate equivalent expressions. These expressions may look different and use different numbers, but the values of the expressions are the same. Provide opportunities for students to experience expressions for amounts of increase and decrease. In Standard 2, the expression is rewritten and the variable has a different coefficient. In context, the coefficient aids in the understanding of the situation. Another example is this situation which represents a 10% decrease: b - 0.10b = 1.00b - 0.10b which equals 0.90b or 90% of the amount.

One method that students can use to become convinced that expressions are equivalent is by substituting a numerical value for the variable and evaluating the expression. For example 5(3 + 2x) is equal to 5 ●3 + 5 ● 2x Let x = 6 and substitute 6 for x in both equations.

$$5(3 + 2 \cdot 6)$$
$$5(3 + 12)$$
$$5(15)$$
$$75$$

$$5 \cdot 3 + 5 \cdot 2 \cdot 6$$
$$15 + 60$$
$$75$$

Provide opportunities for students to use and understand the properties of operations. These include: the commutative, associative, identity, and inverse properties of addition and of multiplication, and the zero property of multiplication. Another method students can use to become convinced that expressions are equivalent is to justify each step of simplification of an expression with an operation property.

Explanations and Examples:

This is a continuation of work from 6th grade using properties of operations and combining like terms. Students apply properties of operations and work with rational numbers (integers and positive / negative fractions and decimals) to write equivalent expressions.

Examples:

Write an equivalent expression for $3(x + 5) - 2$.

Suzanne thinks the two expressions $2(3a - 2) + 4a$ and $10a - 2$ are equivalent? Is she correct? Explain why or why not?

Write equivalent expressions for: $3a + 12$.

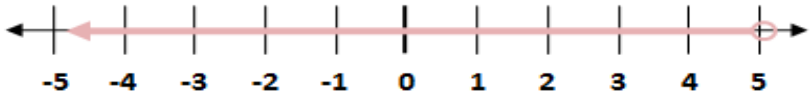
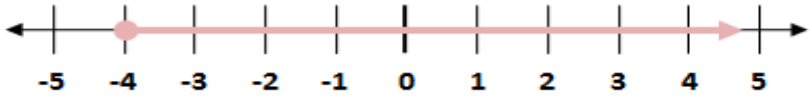
Possible solutions might include factoring as in $3(a + 4)$, or other expressions such as $a + 2a + 7 + 5$.

- A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be $w+w+2w+2w$. Write the expression in two other ways.

Solution: $6w$ or $2(w) + 2(2w)$.

Students apply knowledge gained in learning to combine like terms to solve and graph equations and inequalities. (*Note: the process for solving equations is the same for solving inequalities as shown below.)

Since a solution to an equation would be a single number, the graph would be a closed circle on that number only. However, the graph of an inequality uses a closed circle for ≤ ≥ and an open circle for < > and the line is shaded accordingly.

Math	Number Line Graph
$3x-8<7$ $+8<+8$ $3x < 15$ $\frac{3x}{3} < \frac{15}{3}$ $x < 5$ $(-\infty, 5)$	 Check with 0 (it's pink) $3(0)-8<7$ It works √
$-4m+13\le 29$ $-13\le -13$ $-4m \le 16$ $\frac{-4m}{-4} \ge \frac{16}{-4}$ (notice ≤ changes to ≥) $m \ge -4$ $[-4, \infty)$	 Check with 0 (it's pink) $-4(0)+13\le 29$ It works √

Common Misconceptions and Challenges

Students have trouble interpreting the negative sign simultaneously as “minus” and “negative.” Students may also struggle with operations on negative numbers, having learned procedural rules such as “two negatives cancel each other out.” Memorizing the rules for operations, without sufficient understanding, only undermines students’ abilities to make sense of more advanced concepts.

When adding and subtracting fractions with unlike denominators, students may have trouble rewriting the fractions as equivalent fractions with a common denominator. This process is especially challenging when the fractions include variables.

Students must differentiate between the rules for addition of terms versus the rules for multiplication of terms. Students will commonly misapply the rules for multiplying terms when combining terms with addition.

When using the distributive property, students will often multiply “outside” term by only the first term with the parenthesis . Ex. $4(x + 2) = 4x + 2$, The correct answer would be $4x + 8$.

When solving two-step or multi-step equations, students will often struggle to know which “number” to eliminate from the same side as the variable first. Ex. $3x + 4 = 1$, Student will try to eliminate the 3 rather than start with subtracting 4 from each side.

Students often confuse when to use an open versus a closed circle when graphing inequalities.

Criteria for Success (Performance Level Descriptors)

- **Limited:** Recognize simple equivalent expressions; Solve simple equations. Identify a solution of an inequality
- **Basic:** Apply properties of operations to factor and expand linear expressions with positive integer coefficients; Solve two-step equations with integer coefficients. Solve simple inequalities with positive integer coefficients
- **Proficient**: Apply properties of operations to factor and expand linear expressions with simple rational coefficients. Use variables to create and solve simple equations and inequalities that model word problems
- **Accelerated**: Apply properties of operations t factor and expand linear expressions with rational coefficients; Understand that requiting an expression can show how quantities are related in familiar problem-solving contexts. Construct equations and inequalities with a variable to solve routine problems.
- **Advanced**: Apply properties of operations to factor and expand linear expressions with complex rational coefficients; Understand that rewriting and expression can show how quantities are related in an unfamiliar problem-solving context. Construct equations and inequalities with more than one variable to solve non-routine problems; Use variables to represent and reason with quantities in real-world mathematical situations.

Prior Knowledge	**Future Learning**
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. **6.EE.2**. Write, read, and evaluate expressions in which letters stand for numbers. **6.EE.3**. Apply the properties of operations to generate equivalent expressions. **6.EE.4**. Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them	**8.EE.1** Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **8.EE.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other **8.EE.4**. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities,

Career Connections

Professions that utilize these skills

Advertising, Marketing, promotions, Public relations, and sales managers Computer and information systems managers, Medical and health services managers, Property, real estate, and community association managers, Purchasing managers, buyers, and purchasing age

7.G.1-3

Ohio's Learning Standards – Clear Learning Targets Math 7

7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.

- **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale
- **b.** Represent proportional relationships within and between similar figures

7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions.

- **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Essential Understanding

Scales are another area where ratios and proportions are utilized. They can be greater or less than one.

Students must know how to use a protractor.

Two-dimensional figures can be obtained by slicing through three-dimensional figures.

Students must know the difference between prisms and pyramids.

Vocabulary

- scale drawing
- scale factor
- proportion
- geometric figure
- protractor
- unique
- plane section (cross-section)
- right rectangular prism
- right rectangular pyramid
- edge
- face
- lateral face
- vertices
- polyhedron
- right triangle
- prism
- pyramid

Essential Skills

- I can use ratios and proportions to create a scale drawing.
- I can identify corresponding sides of similar geometric figures.
- I can compute lengths and areas from scale drawings using proportions and scale factors.
- I can reproduce a scale drawing that is proportional to a given geometric figure using a different scale.
- I can determine which conditions create unique triangles, more than one triangle, or no triangle.
- I can construct triangles from three given angle measures or three given side measures to determine when there is a unique triangle, more than one triangle or no triangle using appropriate tools (freehand, rulers, protractors, and technology).
- I can define slicing as the cross-section of a 3D figure.
- I can describe the two-dimensional figures that result from slicing a three-dimensional figure such as a right rectangular prism or pyramid.
- I can analyze three-dimensional shapes by examining two- dimensional cross-sections.

Instructional Strategies

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems.

Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of times you multiple the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations.

Pattern blocks (not the hexagon) provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. Questions about the relationship of the original block to the created shape should be asked and recorded. A sample of a recording sheet is shown.

Shape	Original Side Length	Created Side Length	Scale relationship of created to
Square	1 unit		
Triangle	1 unit		
Rhombus	1 unit		

This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. In other words, if the original side can be stated to represent 2.5 inches, what would be the new lengths and what would be the scale?

Shape	Original Side Length	Created Side Length	Scale relationship of created to
Square	2.5 inches		
Parallelogram	3.25 cm		
Trapezoid	(actual measurements)	Length 1 Length 2	

Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

After students have explored multiple iterations with a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Starting with simple shapes and whole-•-number side lengths allows all students access to discover and understand the relationships. An interesting discovery is the relationship of the scale of the side lengths to the scale of the respective perimeters (same scale) and areas (scale squared). A sample-recording sheet is shown.

Shape	Side Length	Scale	Original Perimeter	Scaled Perimeter	Perimeter Scale	Original Area	Scaled Area	Area Scale
Rectangle	2 x 3	2	10 inches	20 inches	2	6 sq. in.	24 sq. in.	4
Triangle	1.5 inch	2	4.5 inches	9 inches	2	2.25 sq. in.	9 sq. in.	4

Common Misconceptions and Challenges

Student may have misconceptions about:

- Correctly setting up proportions
- How to read a ruler
- Doubling side measures does not double perimeter

Criteria for Success (Performance Level Descriptors)

- **Limited:** Recognize simple geometric shapes based on given conditions; Classify pairs of angles
- **Basic**: Determine a scale from scale drawings of geometric figures and compute and actual length given a measurement in a scale drawing and scale; Draw geometric shapes with given conditions; Determine whether a set of any three given angle or side length measurements can result in a triangle
- **Proficient:** Solve problems involving scale drawing of geometric figures, including computing actual areas from a scale drawing; Using technology or math tools, determine whether a set of any three given angle or side length measures can result in a unique triangle, more than one triangle, or no triangle at all; Identify the two-dimensional figures that result from routine slices of prisms and pyramids
- **Accelerated**: Create and use scale drawings to solve real-world problems; Identify the two-dimensional figures that result from non-routine slices of prisms and pyramids
- **Advanced:** Reproduce scale drawing at a different scale to solve real-world problems; Construct triangles from given conditions that involve a variable

Prior Knowledge

6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = ℓ□w□h and V = B□h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Future Knowledge

8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates).

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them (Include examples both with and without coordinates.)

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Career Connections

Professions that utilize these skills

Engineering and natural sciences managers, Farmers, ranchers, and agricultural manager, Funeral directors Industrial production managers, Medical and health services managers, Forest, conservation, and logging workers, Teachers and Teacher assistants , Art and design occupations, Artists and related workers, Fashion designers, News analysts, reporters, and correspondents, Photographers Writers and editors, Health diagnosing and treating occupations, Optometrists, Registered nurse, Health technologists and technicians, Medical records and health information technicians, Nuclear medicine technology

7.G.4-6

Ohio's Learning Standards – Clear Learning Targets Math 7

7.G.4 Work with circles

a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle.

b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure

7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms

Essential Understanding

Two times the radius of a circle equals its diameter.

Pi is equal to the circumference of a circle divided by its diameter.

Two complementary angles equal a total of 90 degrees and form a right angle.

Two supplementary angles equal a total of 180 degrees and form a straight line.

Use formulas for perimeter, circumference, area, surface area and volume of given polygons and polyhedrons.

Vocabulary

- area
- circumference
- radius
- diameter
- pi
- supplementary
- complementary
- vertical angle
- adjacent angle
- triangles
- quadrilaterals
- polygon
- cube
- right prism

Essential Skills

- I can identify and recognize types of angles: right, supplementary, complementary, vertical, adjacent, and straight.
- I can determine complements and supplements of a given angle.
- I can determine unknown angle measures by writing and solving algebraic equations based on relationships between angles.
- I can determine the formulas for area and volume and then procedure for finding surface area and when to use them in real-world and math problems for two and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.
- I can solve real world and math problems involving area, surface areaand volume of two and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- I can determine the parts of a circle including radius, diameter, area, circumference, center and chord.
- I can identify π.
- I can recognize and apply the formulas for area and circumference of a circle to solve mathematical and real-world problems.
- I can find its circumference, given the area of a circle.
- I can justify that π can be derived from the circumference and diameter of a circle.

Instructional Strategies

This is the students' initial work with circles. Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, pi and area. Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius. Given multiple-•-size circles, students should then explore the relationship between the radius and the length measure of the circle (circumference) finding an approximation of pi an ultimately deriving a formula for circumference. Laying a string or yarn over the circle and compared to a ruler is an adequate estimate of the circumference. This same process can be followed in finding the relationship between the diameter and the area of a circle by using grid paper to estimate the area.

Another visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. In figuring area of a circle, the squaring of the radius can also be explained by showing a circle inside a square. Again, the formula is derived and then learned. After explorations, students should then solve problems, set in relevant contexts, using the formulas for area and circumference.

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations multi-step problems.

Real-•-world and mathematical multi-•-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Common Misconceptions and Challenges

Some students think that π is an exact number rather than understanding that 3.14 isan approximation of pi. Some students also believe that 22/7 is an exact value for π.

Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

Students get confused often between complementary and supplementary angles. It can help to say "It's always *right* to pay a *compliment*". Students could also be taught what supplements are. They are used to make up for items missing from a whole, such as insurance.

Criteria for Success (Performance Level Descriptors)

- **Limited**: Identify the parts of a circle; Calculate the area of triangles and rectangles; Calculate the volume of cubes
- **Basic**: Use supplementary, complementary vertical, or adjacent angles to solve problems with angles expressed as numerical measurements; Calculate the area of quadrilaterals and polygons; Calculate the volume of right rectangular prisms; Calculate the circumference of a circle in mathematical problems
- **Proficient**: Use supplementary, complementary, vertical, or adjacent angles to solve one or two-step problems with angle measurements expressed as variables in degrees; Solve problems involving the area of two-dimensional objects composed of triangles, quadrilaterals, and polygons; Calculate the area and circumference of a circle in real-world and mathematical problems; Solve routine real-world and mathematical problems involving the surface area and volume of three-dimensional objects composed of cubes and right prisms
- **Accelerated**: Use supplementary, complementary, vertical, and adjacent angles to solve multi-step problems with angle measurements expressed as variables in degrees; Given the circumference of a circle, determine its area; Solve real-world and mathematical problems involving the surface area of three-dimensional objects composed of triangles and rectangles
- **Advanced:** Solve problems using formulas for the area and circumference of a circle; Informally describe the relationship between the two measures; Solve complex problems involving the surface area an volume of three-dimensional figures with polygonal faces

Prior Knowledge

6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.5 N/A

6.G.6 N/A

Future Learning

8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.

Career Connections

Professions that utilize these skills

Entertainers and performers, sports and related occupations, Athletes, coaches, umpires, and related workers, Media and communications-related occupations , Writers and editors Health diagnosing and treating occupations, Chiropractors, Optometrists, Registered nurses Veterinarians, Health technologists and technicians, Veterinary technologists and technicians, Protective service occupations Police and detectives, Building and grounds cleaning and maintenance occupations, Grounds maintenance workers, Data entry and information

7.SP.1-2

Ohio's Learning Standards – Clear Learning Targets Math 7

7.**SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population

- **a.** Differentiate between a sample and a population.
- **b.** Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias

7.**SP.2** Broaden understanding of statistical reasoning by using GAISE model

- ***a.*** Formulate questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh grade students compare to the heights of eighth graders?" (GAISE Model, step 1)*
- ***b.*** Collect data: Design and use a plan to collect appropriate data to answer a statistical questions (GAISE Model, step 2)
- ***c.*** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing group to group. (GAISE Model, step 3)
- ***d.*** Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)

Essential Understanding

Recognize sampling techniques such as convenience, random, systematic, and voluntary.

There is no way to survey everyone. Instead, we use samples that represent the population for which we are observing.

Surveys need to be random to ensure a fair survey has been done.

Generate multiple samples (or simulated samples) of the same size to determine the variation in estimates or predictions by comparing and contrasting the samples

Vocabulary

- survey
  population
- sample
- representative
- random sample
- variation
- estimates
- predictions
- understand
- generalizations
- inferences
- generate
- deviation

Essential Skills

- I can define random sample.
- I can identify an appropriate sample size.
- I can analyze and interpret data from a random sample to draw inferences about a population with an unknown characteristic of interest.
- I can generate multiple samples (or simulated samples) of the same size to determine the variation in estimates or predictions by comparing and contrasting the samples.
- I can apply statistics terms such as population, sample, sample size, random sampling, generalizations, valid, biased and unbiased.
- I can recognize sampling techniques such as convenience, random, systematic and voluntary.
- I can recognize that generalizations about a population from a sample are valid only if the sample is representative of that population.
- I can apply statistics to gain information about a population from a sample of the population.
- I can generalize that random sampling tends to produce representative samples and support valid inferences.

Instructional Strategies

In Grade 6, students used measures of center and variability to describe data. Students continue to use this knowledge in Grade 7 as they use random samples to make predictions about an entire population and judge the possible discrepancies of the predictions. Providing opportunities for students to use real-•-life situations from science and social studies shows the purpose for using random sampling to make inferences about a population.

Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample. Have students compare the random sample to population, asking questions like “Are all the elements of the entire population represented in the sample?” and “Are the elements represented proportionally?” Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis.

Provide students with random samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.

Common Misconceptions and Challenges

Students may believe:

One random sample is not representative of the entire population. Many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception

Criteria for Success (Performance Level Descriptors)

- **Limited**: Determine whether a sample is random
- **Basic**: Explain whether s sample is random
- **Proficient:** Describe a random sample of a given population
- **Accelerated**: Use measures of variability for numerical data from random samples to draw informal comparative inferences about multiple populations
- **Advanced:** Use measures of variability for numerical data from random samples to draw informal comparative inferences about multiple populations

Prior Knowledge	Future Learning
6.**SP.1** Develop statistical reasoning by using the GAISE model: a. Formulate Questions: b. Collect Data: c. Analyze Data: d. Interpret Results: **6.SP.2**. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4) **8.SP.2**. Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)

Career Connections

Professions that utilize these skills

Computer operators, Data entry and information processing workers, Desktop publishers, Office and administrative support worker supervisors and managers, Payroll and timekeeping clerks, Tellers Information and record clerks Human resources assistants, except payroll and timekeeping, Material recording, scheduling, dispatching, and distributing occupations, Stock clerks and order fillers, Sales Real estate brokers and sales agents, Travel agents, Life scientists Biological scientists Conservation scientists and foresters, Medical scientist, Architects, surveyors, and cartographers Architects, except landscape

7.SP.3

Ohio's Learning Standards – Clear Learning Targets Math 7

7.SP.3 Describe analyze distributions

a. Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point

b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variables, measuring the difference between the centers by expressing it as a multiple of a measure of variability*. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable*

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Essential Understanding

Students need to remember the definitions of mean, median, and mode.

Variability is responsible for the overlap of two data sets and that an increase in variability can increase the overlap, and

Median is paired with the interquartile range and mean is paired with the mean absolute deviation.

Vocabulary

- visual overlap
- data distribution
- variability
- measure of variability
- measures of center
- comparative inferences
- population
- assess
- measuring
- use
- draw inferences

Essential Skills

- I can find measures of central tendency (mean, median, and mode) and measures of variability (range, quartile, etc.).
- I can analyze and interpret data using measures of central tendency and variability.
- I can draw informal comparative inferences about two populations from random sample.
- I can identify measures of central tendency (mean, median, and mode) in a data distribution.
- I can identify measures of variation including upper quartile, lower quartile, upper extreme-•-maximum, lower extreme minimum, range, interquartile range, and mean absolute deviation (i.e. box-•-and-•-whisker plots, line plot, dot plots, etc.).
- I can compare the differences in the measure of central tendency in two numerical data distributions by measuring the difference between the centers and expressing it as a multiple of a measure of variability.

Instructional Strategies

In Grade 6, students used measures of center and variability to describe sets of data. In the cluster "Use random sampling to draw inferences about a population" of Statistics and Probability in Grade 7, students learn to draw inferences about one population from a random sampling of that population. Students continue using these skills to draw informal comparative inferences about two populations.

Provide opportunities for students to deal with small populations, determining measures of center and variability for each population. Then have students compare those measures and make inferences. The use of graphical representations of the same data (Grade 6) provides another method for making comparisons. Students begin to develop understanding of the benefits of each method by analyzing data with both methods.

When students study large populations, random sampling is used as a basis for the population inference. This build on the skill developed in the Grade 7 cluster "Use random sampling to draw inferences about a population" of Statistics and Probability. Measures of center and variability are used to make inferences on each of the general populations. Then the students have make comparisons for the two populations based on those inferences.

This is a great opportunity to have students examine how different inferences can be made based on the same two sets of data.

Common Misconceptions and Challenges

Students often get mean, median, and mode mixed up. Remind students that the mean is the average. The words are interchangeable. The median is similar to a median down the road: it splits data into two equal groups. The mode is the one that occurs most often. Also, there can be more than one mode, or there could be no mode.

Criteria for Success (Performance Level Descriptors)

- **Limited:** Use the mean to compare and draw inferences about two different populations
- **Basic:** Use measures of center to draw comparisons about two different populations
- **Proficient:** Use measures of variability to draw comparisons about two different populations
- **Accelerated:** Use measures of variability for numerical data from random samples to draw informal comparative inferences about two populations
- **Advanced**: Assess the degree of visual overlap of two numerical data distributions with similar variability

Prior Knowledge	**Future Learning**
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. Summarize and describe distributions	**6.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Career Connections

Professions that utilize these skills

Aerospace engineers, Biomedical engineers, Chemical engineers Computer hardware engineers Electrical engineers, Management occupations Computer and information systems managers, Farmers, ranchers, and agricultural managers, Financial managers Industrial production managers, Medical and health services managers, Purchasing managers, buyers, and purchasing agents Information and record clerks, Human resources assistants, Farming Forest, conservation, and logging workers, Metal workers and plastic workers, Computer control programmers and operators, Business and financial operations occupations, Budget analysts

7.SP.5-8

Ohio's Learning Standards – Clear Learning Targets Math 7

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-•-run relative frequency, and predict the approximate relative frequency given the probability.

7.**SP.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- **a.** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
- **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

7.**SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
- **c.** Design and use a simulation to generate frequencies for compound events.

Essential Understanding

Probability can be expressed as decimals, fractions, and percentages. Students need to know that it is possible to have a percentage greater than 100%.

Students recognize that the probability of any single event can be can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1

Students must know that a probability of 0 or 0% indicates an impossibility.

Students should be able to conduct experiments and make predictions based on their outcome.

Students should understand compound events.

Vocabulary

- Probability
- chance event
- chance process
- sample space
- tree diagram
- probability model
- simulation
- compound events
- frequencies
- predict
- design
- generate
- probabilities

Essential Skills

- I can determine relative frequency (experimental probability) is the number of times an outcome occurs divided by the total number of times the experiment is completed.
- I can determine the relationship between experimental and theoretical probabilities by using the law of large numbers.
- I can predict the relative frequency (experimental probability) of an event based on the (theoretical) probability.
- I can use models to determine the probability of events.
- I can identify the outcomes in the sample space for an everyday event.
- I can define and describe a compound event.
- I can know that probability is expressed as a number between 0 and 1.
- I can understand that a random event with a probability of ½ is equally likely to happen and not happen.
- I can understand that as probability moves closer to 1 it is increasingly likely to happen and as probability moves closer to 0 it is decreasingly likely to happen.
- I can draw conclusions to determine that a greater likelihood occurs as the number of favorable outcomes approaches the total number of outcomes.
- I can recognize uniform (equally likely) probability.

Instructional Strategies

Grade 7 is the introduction to the formal study of probability. Through multiple experiences, students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions.

Help students understand the probability of chance is using the benchmarks of probability: 0, 1 and ½. Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as 1/2. Then advance to situations in which the probability is somewhere between any two of these benchmark values. This builds to the concept of expressing the probability as a number between 0 and 1. Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen. Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that the more the
simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data; i.e., a sample space of the compound event. From each sample space, students determine the probability or fraction of each possible outcome. Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability.

Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities.

Common Misconceptions and Challenges

Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same.

Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities. Note examples in simulations when some possibilities are not shown.

Criteria for Success (Performance Level Descriptors)

- **Limited:** Understand that probabilities are numbers between 0 and 1
- **Basic:** Find probabilities in straightforward situations
- **Proficient:** Understand that a probability near 0 indicates an unlikely event, a probability near ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event; Compare theoretical and experimental results for a probability experiment
- **Accelerated**: Find probabilities of compound events in a real-world context; Use example situations to explain the differences between theoretical and experimental probabilities
- **Advanced:** Explain why events are likely or unlikely and use this explanation to make predictions; Develop a probability model and use it to find probabilities of events; Compare theoretical probabilities (from a model) to observed frequencies (experimental); explain possible sources of the discrepancy between the two measures

Prior Knowledge	**Future Learning**
6.SP.5 Summarize numerical data sets in relation to their context. share. Find measures of variability (range and interquartile range) as	**N/A**

Career Connections

Professions that utilize these skills

Administrative services managers , Computer and information systems managers , Engineering and natural sciences managers , Farmers, ranchers, and agricultural managers, Financial managers Funeral directors , Computer and mathematical occupations, Actuaries Computer programmers, Computer software engineers, Mathematicians Statisticians, Architects, surveyors, and cartographers, Surveyors, cartographers, photogrammetrists, and surveying technicians Information and record clerks, Human resources assistants, except payroll and timekeeping, Material recording, scheduling, dispatching, and distributing occupations Stock clerks and order fillers, Other office and administrative support occupations, Data entry and information processors.